NORSMA 8 - Extended summaries of accepted papers

(alphabetical on surname)

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The characteristics of formative assessment that enhance student achievement in mathematics

Introduction

Several research reviews have demonstrated that formative assessment can substantially improve student achievement (e.g. Black & Wiliam, 1998; Hattie, 2009), and individual studies specify the benefits for low achieving students in mathematics (e.g. Kirton, Hallam, Peffers, Robertson & Stobart, 2007). From its strong research base, the use of formative assessment is recommended in mathematics education (National Mathematics Advisory Panel, 2008) and for students with special educational needs (The European Agency for Special Needs and Inclusive Education, 2007). However, research characterizing what formative assessment to include in professional development and how to support teachers' implementation of such a practice is lacking (Schneider and Randel 2010).

For purposes of gaining valuable insights about best practices it is important that implementations of formative assessment, that are empirically linked to student achievement in scientific studies, are carefully analysed and described. These analyses may provide information about specifics of such practices as well as how these specific characteristics may have functioned as part of an enhanced learning process.

The aim of the present study was to characterize the kind of formative assessment that came out as a result of a professional development program in formative assessment, a program that had showed significant effect on student achievement in mathematics in comparison with a control group ($p = 0.036$, $d = 0.66$), using a pre-test and a post-test measuring both procedural and conceptual understandings.

Formative assessment

The study uses a resently established framework of formative assessment comprising the big idea of using evidence of student learning to adjust instruction to better meet the needs of the students, and the following five key strategies (Wiliam & Thompson, 2008):

1. clarifying, sharing and understanding learning intentions and criteria for success

2. engineering effective classroom discussions, questions, and tasks that elicit evidence of learning

3. providing feedback that moves learners forward

4. activating students as instructional resources for one another

5. activating students as the owners of their own learning

This framework emphasises that formative assessment may pertain to, and be inherent in, the whole classroom practice rather than separate activities conducted by different individuals. Thus, it constitutes an overarching description of the idea of formative assessment as a unified practice of integrated strategies.

Methodology

The study analyses the changes in the mathematics classroom practice made by a random selection of twenty-two Year 4 teachers due to a professional development program in formative assessment. The framework of formative assessment described above structured the content of the professional development program, the data collection and the data analysis.

Two classroom observations and one teacher interview, for each teacher, were conducted to unveil the new formative assessment activities regularly used in the mathematics classroom practice after the professional development program.

The analysis first identified and listed formative assessment activities for each teacher, which then were complied into a list of activities most commonly implemented on a group level. Further analysis was made by using comprehensive narratives set up for each teacher's changes, structured along the five key strategies and the big idea.

Findings

The teachers' changes span from complementing previous teaching with new activities that enhance the big idea of formative assessment to a classroom practice that is radically developed in its very foundation. It was found that the teachers had not just added new formative assessment activities; they used those activities in line with the intended function. Common changes among the teachers were an extended repertoire of eliciting evidence of learning from all students. Receiving more frequent and qualitative information, the teachers could adjust their mathematics instruction to better meet students' learning needs, either in a new way, in a modified way, or in the same way as before the intervention but potentially more often.

Discussion

The study shows that it is possible through a professional development program to support a random selection of teachers to implement a formative assessment practice, characterised as a unity of integrated formative assessment strategies, that significantly improves student achievement.

The changes the teachers made, and thus the characteristics of their new formative classroom practice, can be described in relation to three dimensions of formative assessment, which are suggested to afford new opportunities for student learning. First, an integration of three key processes of teaching and learning may enhance student learning (where the learner is going, is right now and the next step in learning). A second dimension indicates that further learning opportunities may occur by involving all agents (teacher, student, and peers) in these processes. Lastly, shortened adjustment cycles make the formative assessment more time efficient.

Formative assessment can be justified from its potential to point teachers towards specific learning problems that need to be addressed in the mathematics classroom practice (Ginsburg, 2009), and to strengthen an inclusive learning environment (The European Agency for Special Needs and Inclusive Education, 2007). This study provides implications for what formative assessment to include in teacher training. However, the characterisation of formative assessment as a unity of integrated formative assessment strategies also show how complex this practice is, which implicates the need of major support to the teachers to successfully implement such a practice.

References

Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education: Principles, Policy & Practice, 5*(1), 7-74.

Ginsburg, H. P. (2009). The Challenge of Formative Assessment in Mathematics Education: Children's Minds, Teachers' Minds. *Human Development 52*, 109–128.

Hattie, J. (2009). *Visible learning: a synthesis of over 800 meta-analyses relating to achievement.* London: Routledge.

Kirton, A., Hallam, S., Peffers, J, Robertson, P., & Stobart, G. (2007). Revolution, Evolution or a Trojan Horse? Piloting Assessment for Learning in Some Scottish Primary Schools. *British Educational Research Journal 33*(4), 605-27.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

Schneider, M. C., & Randel, B. (2010). Research on characteristics of effective professional development programs for enhancing educators' skills in formative assessment. In H. L. Andrade & G. J. Cizek (Eds.), *Handbook of formative assessment* (pp. 251-276). Abingdon: Routledge.

The European Agency for Special Needs and Inclusive Education. (2007). *Assessment in Inclusive settings.* Retrieved 2015-10-07 from https://www.european-agency.org/agency-projects/assessment-in-inclusive-settings

Wiliam, D., & Thompson, M. (2008). Integrating assessment with learning: what will it take to make it work? In C. A. Dwyer (Ed.), *The Future of Assessment: Shaping Teaching and Learning* (pp. 53-82). Mahwah, NJ: Lawrence Erlbaum Associates.

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Qual(Equ)ity and Legitimacy
In connection with National Testing in Sweden.

Introduction: This paper discusses some findings from my doctoral dissertation *Is school for everyone? The national test in mathematics at Grade three in Sweden.* [1]In this thesis, I understand the national test as an arena for equity and conclude that the test raises issues of legitimacy with an enhanced focus on achievement. Legitimacy is understood following Lundahl and Tveit (2015) as making something or someone justified, righteous or accepted.

During implementation of the national tests in Mathematics in the third grade in year 2010, a double purpose of the test was stated - to monitor the national quality of education as well as the individual level of knowledge (Björklund, Boistrup & Skytt, 2011; Ministry of Education and Research, 2012; The Swedish Government, 2006). I mean that when pupils' scores in this test is connected to quality of education,high scores are connected to high quality of education. Following this, low achievements carries the risk of putting the school's legitimacy as an educational institution into question. Lundahl and Tveit (2015) discuss how the double purpose of the national test threatens not only the legitimacy of the test but also the legitimacy of teachers as professionals. I understand the national tests to also be conducted at a time when school policy is influenced by management through objectives or New Public Management (NPM). There is a neo-liberal approach also at play where decentralization and freedom to choose between alternatives are central (Antikainen, 2006; Telehaug Mediås, & Aasen, 2006). Such governance may also lead to dominance of market values like consumer choice and competition instead of values like equality and social justice (Hudson, 2011).

Purpose: The aim of this paper is to investigate the discourse on school legitimacy and its connection with concepts of achievement and equity in teachers' talk in relation to the national tests. The three research questions are: Is school and teacher legitimacy talked about, if so, how; Is quality talked about, if so, how?; Is equity talked about, if so how?

Theory: Foucaults theories (Faubion 1994; Foucalt, 1989) where discourse is described as representations of knowledge, truth and power govern what is possible to talk about, for who and when. Discourse recreated and constructed as these systems of representations are put to use by individuals (Hall, 2001). Positioning of individuals is further understood as socially accepted ways of talking and acting (Gee, 2008)

[1] With a background in the project 'What does testing do to pupils', together with Gunnar Sjöberg, Eva Silfver and Mikaela Nyroos, at Umeå University, is financed by the Swedish Research Council.

governed in turn by discourses which create knowledge that individuals are carriers of (Davies & Harré, 2001).

Methodology: The data in this article draws on two articles in my thesis (Bagger, 2015b; Preprint). The data was collected through interviews with eight teachers and 102 pupils at three different schools, during field work in the academic year 2010-2011. The analytical method was discourse and positioning analysis of the pupil's and teacher's talk. The positionings of the pupil, teacher and school were identified and analyzed in the talk, as well as the connection of these positions to talk about achievement and equity.

Results: My findings from analysis of data reveal that a discourse of legitimacy is activated in relation to talk about tests. This discourse connects knowledge about the quality of the pupil's knowledge during the test and the quality of education in relation to pupil's scores. Equity is talked about as affecting the pupil's scores and having a bearing on the teacher's professional legitimacy and being supportive in situations where support is needed, as also pupil's legitimacy as future learners and test takers. The talk about "straightening up" during tests are important statements in this regard. These point to the idea that focusing and seriousness on tests should lead to higher scores as well as display a school offering higher levels of quality of education. Equity is not mentioned in these contexts. In the teacher positioned as a test-taker his or her pupil's equity is less important. The legitimacy of the school becomes more important than pupil's achievement. An important finding here is that teacher's legitimacy as a test-taker might be at risk if the support they give to pupils can in one way or another be judged as inappropriate.

Discussion: Some conclusions regarding the legitimacy of school discourse are:

1. It makes the school balance the pupil's equity against the test's equality.
2. Teacher's and school's legitimacy is two-fold and could be at stake in two different ways depending on the teacher's positioning: the positioning of being a supportive teacher and the position of being a controlling test-giver.
3. Even pupil's legitimacy is put at stake, which is unfortunate since pupils are supposed to be taught and approached accordingly to his or her individual needs.

I suggest that it is possible to combine the educational goals of quality and equity in order to promote the quality in equity and at the same time equity of quality. For this purpose I would like to suggest and elaborate a new concept, namely the Qual(Equ)ity of results and effects of tests. This word implies that achievement and support during tests to pupils, need to be considered simultaneously and that these two are intertwined. This could possibly shift the focus during tests from a controlling emphasis presently found to students' learning.

This paper contributes to wider discussion regarding how to approach tests when they are carried out, when they are evaluated and how the results are used, enabling pupils, teachers and schools to focus on learning during tests rather than test taking.

References:

Antikainen, A. (2006). In Search of the Nordic Model in Education. *Scandinavian Journal of Educational Research, 50*(3), 229-243.

Bagger, A. (2015a). Prövningen av en skola för alla. Nationella provet i matematik i det tredje skolåret. Doktorsavhandling, Umeå Universitet.

Bagger, A. (2015b). Pupil's equity vs the test's equality? Support during third graders' national tests in mathematics in Sweden. *CURSIV.* Copenhagen: Danish School of Education, Aarhus University. (Preprint).

Bagger, A. (Preprint). Pressure at stake. Swedish third graders' talk about national tests in mathematics. *Nordic Studies in Mathematics Education.*

Björklund Boistrup, L., & Skytt, A. (2011). Ämnesprovet i årskurs 3 (pp. 255- 262): Matematik - ett grundämne Göteborg : National center for mathematics education (NCM).

Faubion, D., J. (Ed.) (1994). Power. The Essential Works of Foucault, 1954-1984, vol 3. Penguin Books: London

Foucault, M., (1989). Power/knowledge- selected interviews and other writings 1972-1977. Gordon, C., (ed).

Foucault, M., Bjurström, C. G., & Torhell, S.-E. (2011). *Vetandets arkeologi*. Lund: Arkiv.

Hall, S. (2001). Foucault, Power, Knowledge and Discourse. In: M. Wetherell, S. Taylor & S.J. Yates (Eds.), *Discourse theory and precice: a reader* (pp. 72-81). London; Sage.

Hudson, C. (2011). Evaluation: the (not so) softly-softly approach to governance and its consequences for compulsory education in the Nordic countries. *Education Inquiry*, *2*(4), pp. 671-687.

Lundahl, Christian & Tveit, Sverre (2015). Att legitimera nationella prov i Sverige och i Norge – en fråga om profession och tradition. *Pedagogisk forskning i Sverige.*

Ministry of Education and Research. (2012). *Fler nationella prov i vår.* Press release, 30 October 2008. Retrieved 20130901 from The Ministry of Education and Research.

The Swedish Government. (2006). *Uppdrag till Statens skolverk att föreslå mål att uppnå och nationella prov i årskurs 3*. (U2006/8951/S).

Telehaug, A,O., Mediås, O.A., & Aasen; A. (2006) The Nordic Model in Education: Education as Part of the Political System in the Last 50 Years*. Scandinavian Journal of Educational Research*, *50*(3) pp. 245-283.

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Relationen mellem matematiklæreres opfattelse af elever med særlige behov og lærernes praksis

Projektets forskningsspørgsmål

Projektet har sit afsæt i følgende forskningsspørgsmål:
Hvilke opfattelser og begreber har matematiklærere om elever med særlige behov i matematikholdige situationer og hvilke relationer er der mellem disse opfattelser og begreber og matematiklærernes praksis?

Metode

Empirien indeholder beskrivelser af 18 elever (0. - 5. klasse) skrevet af 18 matematiklærere fra 10 forskellige danske folkeskoler. Hver lærer har beskrevet en elev fra egen klasse, som læreren vurderer har *særlige behov* i matematikholdige situationer. De 18 lærere har efterfølgende deltaget i ét semikonstrueret gruppeinterview. I alt har der været afholdt fire gruppeinterview med henholdsvis 3, 5, 5 og 5 lærere. Formålet med interviewene har været at få lærerne til at udfolde deres fortællinger og beskrivelse af deres valgte elev og dennes situation set i forhold til deres praksis.

Gruppeinterviewet er designet på baggrund af en analyse af lærernes skriftlige beskrivelser af eleverne. Under interviewet fortæller lærerne først deres gruppe om deres udvalgte elev. Herefter bliver lærerne præsenteret for fire forklaringsmodeller, som forskningen peger på kan være årsag til, at elever har vanskeligheder i skolefaget matematik: medicinsk/neurologisk, psykologisk, sociologisk og didaktisk (Engström. 2013). Lærerne får herefter mulighed for at svare på og begrunde, hvilke forklaringsmodeller, de mener, kan ligge til grund for elevens situation og hvilke tiltag de mener, de kan sætte i gang for at hjælpe eleven videre i elevens læringsproces, hvis årsagen kan findes inden for den/de forklaringsmodeller. Læreren får også mulighed for at svare på, hvilke udfordringerne de oplever, de står overfor ud fra dette perspektiv.

For at få svar på hvilke relationer der er mellem matematiklæreres opfattelser og begreber af *elever med særlige behov* og lærernes praksis, er der på baggrund af de foretagne gruppeinterviews foretaget en samlet analyse af a) lærernes valg af forklaringsmodeller og begrundelserne derfor, b) de udfordringer lærerne oplever, at de står overfor i mødet med eleverne og 3) de tiltag lærerne har iværksat eller eventuelt vil iværksætte med henblik på at hjælpe eleverne videre i deres læringsproces.

Resultater

Resultat af analysen viser, at 12 af de 18 lærere som 1. prioritet vælger enten den medicinsk/neurologiske eller den psykologiske forklaringsmodel for elevens situation. Kun to lærere vælger den didaktiske forklaringsmodel. På baggrund af resultaterne kan der konkluderes, at det at vælge den didaktiske forklaringsmodel i en kollegial samtale kræver mod og selverkendelse. Analysen viser videre, at som lærerne reflekterer over elevens situation i relation til egen praksis får lærerne øje på flere didaktiske forklaringer, som kan ligge til grund for, at eleven er kommet, er eller har risiko for at fortsætte med at være i kategorien "elever med særlige behov i matematikholdige situationer. " Afslutningsvis konkluderer projektet, hvilke fagdidaktiske kompetencer det kan være relevant, at matematiklærere videreudvikler i egen praksis i arbejdet med at hjælpe *elever med særlige behov* videre i deres læringsproces.

Diskussion

Jeg ønsker at diskutere projektets resultater, herunder interviewguidens design og anvendelighed i arbejdet med at udvide læreres fortællinger og fremme deres refleksioner over *elever med særlige behov* med det formål, at lærerne får øje på, hvor det kan være relevant at udvikle deres praksis med målet at give alle elever et fagligt løft inden for faget matematik, men også et ønske om, at nærme os en reel inklusion af alle elever i undervisningen.

References

Di Martino P., Zan R. (2010). 'Me and Math': toward a definition of attitude grounded on students´ narratives. Journal of Mathematics Teacher Education, vol. 13, n.1, pp. 27-48.

Engström, A. (2013). Matematikvanskeligheder - nogle grundlæggende problemstillinger. I: Weng & Andersen, W. (red.) *Håndbog for matematikvejledere*. Dansk Psykologisk Forlag

Ernest, P. 2011: Mathematics and Special Educational Needs: Theories of mathematical ability and effective types of intervention with low and high attainers in mathematics, Lambert Academic Publishing

Hedegaard-Sørensen, L. (2010). Pædagogiske og didaktiske rum for elever med diagnosen autismespektrumforstyrrelse: Om læreres selvforståelse og handling I (special) pædagogisk praksis. Ph.d.-afhandling, København: Danmark Pædagogiske Universitetsskole, Aarhus Universitet.

Heyd-Metsuyanim, E. (2013). The Co-Construction of Learning Difficulties in Mathematics-Teacher Student Interactions and Their Role in the Development of a Disabled Mathematical Identity. Educational Studies in Mathematics, 83(3), 341-368)

Lave, J. & Wenger, E. (2003). Situeret læring (13-103). I J. Lave & E. Wenger, Situeret læring - og andre tekster. København: Hans Reitzel

Leung, K. S. F. (2006). Mathematics education in East Asia and the west: Does culture matter? In Leung, Frederick K.-S., Graf, Klaus-D., Lopez-Real, Francis J. (Eds.). Mathematics Education in Different Cultural Traditions - A Comparative Study of East Asia and the West. (pp. 21-46) The 13th ICMI Study. New York: Springer.

Magne, O. (2006). Historical aspects on special education in mathematics. Nordisk matematikk didaktikk, (11) 4, 7-36.

Schmidt, M. C. S. (2013). Klasseledelse i matematik. Hvad ved vi egentlig? - Et systematisk review om matematiklæreres bidrag til et inkluderende læringsfællesskab på skolernes begynder- og mellemtrin. Mona 2013 (3) 23-43.

Sjöberg, G. (2003). Dyskalkuli, skolans största pedagogiska problem? En granskning af forskningslitteraturen mellem 1993-2003 (261-282). I Engstöm, A. (red.), Democracy and Participation A Challenge for Special Needs Education in Mathematics. Ôrebro: Örebro University, Department of Education, Forskningskollegiet.

Schleppenbach, Meg; Flevares, Lucia M.; Sims, Linda M.; Perry, Michelle (2007). Teachers' Responses to Student Mistakes in Chinese and U.S. Mathematics Classrooms. *The Elementary School Journal*, v108 n2 p131-147.

Undervisningsministeriet – Styrelsen for It og Læring [The Ministry of Education in Denmark - Agency for ICT and Learning] (2014). Matematik – Mål, læseplan og vejledning. [Mathematics – Goals, curriculum and guidelines] Avaiable April 2015 at www.emu.dk/modul/matematik-mål-læseplan-og-vejledning

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Elever med låga prestationer i matematik – bakgrund och orsaker

SYFTE

Syftet med studien som ingår i ett avhandlingsprojekt är att genom litteraturanalys av tidigare forskning samt intervjuer av lärare och elever redovisa förklaringar till uppkomsten av matematiksvårigheter. Studien syftar till att ur ett utbildningsvetenskapligt perspektiv synliggöra förklaringar till låga prestationer i matematik som inte har ett neurofysiologiskt ursprung. Det kan då vara fråga om exempelvis brister i undervisningen, oroliga arbetsför- hållanden eller annan påverkan av elevens sociala omgivning och miljö. Dessa orsaker kan bli tillgängliga för de aktörer som utvecklar specialpedagogiska metoder för undervisningen av elever med låga prestationer i matematik. Förutom detta övergripande syfte finns följande delmål med studien:

För det första undersöks mängden av de elever som enbart har matematiksvårigheter, den grupp som benämnes specifik SUM och därmed ej klarar godkänt i ämnet matematik i skolår 7, 8 och 9 i elva kommuner i nordvästra Skåne. Utöver detta undersöks hur många elever som ej har godkänt i något av de övriga ämnena. Detta för att få en bild av om det är fler som inte klarar matematiken än de övriga ämnena. Eftersom termen *Särskilt utbildningsbehov i matematik (SUM)* står för alla elever som inte når målet godkänd i matematik finns bland dessa elever många som inte når de uppsatta målen även i andra ämnen. Det kan då handla om allmänna svårigheter med olika förklaringar till problemen i ämnet. Även bland de elever som har icke godkänt endast i matematik och därmed tillhör gruppen *Specifikt särskilt utbildnings- behov i matematik (Specifik SUM)* och sålunda har icke godkänt endast i matematik finns det med all sannolikhet elever med helt olika förklaringar till att de fått svårigheter med matema- tiken. Genom att intervjua några elever ur såväl gruppen med SUM som gruppen med specifik SUM belyses de bakgrundsbetingelser som ligger bakom elevernas prestationer. Lärarintervjuerna förväntas klargöra de åtgärder som skolan har vidtagit för att stödja eleven i strävan att bi godkänd i matematik. Avsikten är också att se om det i den internationella forskningslitteraturen finns angivet några speciella förklaringar till låga prestationer i övriga ämnen förutom matematiken.

METOD

Studien omfattar tre delar med olika forskningsmetoder. I den första avdelningen ges en teoretisk bakgrund till begreppet dyskalkyli, och en analys av hur detta begrepp och olika kognitiva svårigheter är vetenskapligt förankrade i den internationella forskningslitteraturen.
Följande förklaringsmodeller har blivit föremål för litteraturanalys: medicinska/neurologiska, psykologiska, sociologiska och didaktiska. De är beteckningar för förklaringar till uppkomst av låga prestationer i matematik och även andra ämnen i skolans undervisning. Ett 60-tal olika studier företrädesvis från internationell forskning utgör underlag för litteraturanalysen.

Den andra delen omfattar en kvantitativ studie som kommer att inkludera officiella uppgifter om hur många elever i årskurs 7 till 9 i elva kommuner som inte når godkänt (betyget E) i matematik under tre läsår. Dessutom samlas in uppgifter om hur många elever som icke når godkänt enbart i något av ämnena svenska, engelska eller något av de natur- eller samhällsorienterande ämnena. Vid betygsinventeringen noteras om eleven har icke godkänt i såväl matematik som något övrigt ämne eller icke godkänt enbart i matematik. Även betyg på nationellt prov i matematik i skolår nio noteras för dessa elever.

Den tredje delstudien omfatta en kvalitativ studie med semistrukturerade intervjuer med matematiklärare för de elever som inte uppnått godkänt i årskurs 8 samt ett antal av dessa elever när de går i årskurs 9. Det är även viktigt att identifiera elevens egen syn på vad som ligger bakom det låga resultatet i matematik. Dessutom får eleverna redogöra för sitt eget förhållningssätt till den undervisning de fått i ämnet matematik. Genom att intervjua några elever ur såväl gruppen med SUM som gruppen med specifik SUM är avsikten att kunna belysa de bakgrundsbetingelser som ligger bakom elevernas prestationer samt deras egna förklaringar.

RESULTAT

Litteraturanalys

Dyskalkyli är en biologiskt influerad avvikelse som kännetecknas av svårigheter att lära och tillämpa matematik genom hela livet. En knapphändig forskning har medfört svårigheter att ge en enhetlig definition av begreppet (Mazzocco & Räsänen 2013). Begränsningar i arbetsminnets kapacitet är associerade med inlärninssvårigheter i matematik (Ashcraft & Kirk 2001). Ett speciellt begrepp har skapats för oro, rädsla och stress inför matematiklektioner, nämligen matematikängslan. Denna ängslan kan leda till att elever kan hamna i en situation med ständigt låga prestationer (Dowker 2005). Brister i undervisningen kan vara en starkt bidragande orsak till uppkomsten av matematiksvårigheter (Sjöberg 2006). Sociokulturella faktorer, exempelvis föräldrarnas utbildning och kulturella kapital får allt större betydelse för elevernas resultat i skolan (Considine & Zappalà 2002).

Betygsinventering

Eftersom denna empiri ännu inte är klar hänvisas till följande hypotes:

Tidigare forskning (Magne 2006) har kommit fram till att antalet elever med specifik SUM kan vara så lågt som 1 % av samtliga elever i den aktuella årskursen. Jag utgår därför ifrån att antalet elever med specifik SUM i skolår 9 i min undersökning är mycket lågt, någon eller några procent av det totala antalet elever, vars betyg jag inhämtat. Om denna hypotes kan verifieras, framstår de bedömningar som görs angående andelen elever med dyskalkyli, oftast mellan 5-6 %, som höga.

Intervjuer

I elevsvaren framkommer följande förklaringar till deras låga prestationer i matematik: låga arbetsinsatser, matematikängslan, svårigheter att förstå ämnet, täta lärarbyten, stökig arbetsmiljö och bristande undervisning. Dessa förklaringar kan i huvudsak relateras till elevernas sociala omgivning som har gett upphov till de problem som ligger bakom elevernas låga resultat i matematik. Även olika undervisningsstrategier kan ha betydelse för elevernas matematiska utveckling.

De intervjuade lärarna förklarar elevernas låga prestationer med att visa elever har dåliga förkunskaper, är ointresserade och presterar låga arbetsinsatser. I vissa fall har sociala svårigheter i hemmet initierat problemen. I endast ett fall bedömer läraren att eleven har dyskalkyliska svårigheter.

STUDIENS RELEVANS

Betraktar vi låga prestationer i matematik som ett pedagogiskt problem är det viktigt att vi inom ramen för ett matematikdidaktiskt arbete undersöker bakgrund och förklaringar till elevernas matematikproblem och anvisar pedagogiska åtgärder för att minska dessa. Problemet behöver få den uppmärksamhet som är berättigad av dess konsekvenser för en stor grupp elever i vårt skolsystem. Det saknas även evidens för metoder som används i arbetet med elever som har låga prestationer i matematik. I denna diskussion om orsakerna till svårigheterna och hur skolan kan hjälpa elever som har problem med matematiken kan den empiriska utbildningsvetenskapliga forskningen spela en mer framträdande roll än som hittills varit fallet.

REFERENSER

Alro, H., Skovsmose, O., & Valero, P. (2005). Researching multicultural mathemathics classroom through the lens of landscapes of learning. *Aalborg University.* Hämtat från www.dpu.dk/Everest/Publications/Medarbejdere/

Ashcraft, M. H., & Kirk, E. (2001). The relationships among working memory, math anxiety
and performance. *Journal of Experimental Psychology, 130*(2), 224-237.

Beilock, S., Gundersson, E., Ramirez, G. and Levine, S. (2010). Female teachers math anxiety affects girls`math achievement. *Proceedings of the National Academy of Sciences of the United States of America,* 107(5): 1860-1863.

Berch, D., & Mazzocco, M. (2007). *Why Is Math So Hard for Some Children?* Baltimore: Md.: Paul H. Brookes Pub. Co., cop.

Considine, G., & Zappala, G. (2002). Factors Influencing the Ducational Performance of
Students from Disadvantaged Backgrounds, in Eardley, T., & Bradbury, B. (2002).

(red). Competing Visions, Proceedings of the National Social Policy Conference, Sydney, 4-6 July 2001. *SPRC Report 1/02, University of New South Wales, Sydney, NSW, 2052, Australia.*

Dowker, A. (2005). *Individual differences in Arithmetic. Implications for Psychology, Neuroscience and Education.* New York: Psychology Press.

Engström, A. & Magne, O. (2003*). Medelsta-matematik II – Hur väl behärskar grundskolans elever lärostoffet enligt Lgr 69, Lgr 80 oc h Lpo 94?* Örebro: Örebro universitet, Pedagogiska institutionen.

Magne, O. (1998). *Att lyckas med matematik i grundskolan*. Lund: Studentlitteratur.

Magne, O. (2006). Historical Aspects on Special Education in Mathematics. *Nordic Studies In Mathematics Education, Volume 11,* 7-34.

Mazzocco, M. M. M., & Räsänen, P. (2013). Contributions of longitudinal studies to evolving definitions and knowledge of developmental dyscalculia. *Trends in Neuroscience & Education, 2*(2), 65.

Sjöberg, G. (2006). *Om det inte är dyskalkyli – vad är det då?* Doktorsavhandling. Umeå: Umeå Universitet.

Vygotskij, L. (1978). *Mind in Society. The developement of Higher Psychological Processes.* Eds. Cole et al. Cambridge: Harvard University Press.

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Enhancing student learning and motivation in mathematics with computer-assisted instruction in vocational upper secondary education

Aims

The aim of this study is to investigate the effects of a computer program, MinecraftEdu on Finnish vocational upper secondary students. More specifically we will test if computer assisted intstruction with MinecraftEdu has an effect on students': 1) mathematical skill development 2) mathematics interest 3) mathematics self-concept 4) mathematics anxiety.

Methodology

Participants. The study was carried out in a large city in Southern Finland. The experimental group consisted of fourteen Swedish-speaking second-year vocational upper secondary. The control group consisted of fourteen Swedish-speaking second-year students from the same secondary school.
Methods of data collection. Students' mathematical skills were assessed with the RMAT-test (Räsänen, Linnanmäki, Haapamäki, & Skagersten, 2008), and ten self-developed items on geometry and algebra. Mathematics interest was measured with seven items based on questionnaires from studies by Frentzel et al. (2012) and Renninger and Su (2012). Students' mathematics self-concept was measured with 10 items from the Students Self Description Questionnaire III (Marsh, 1984). The Abbreviated Math Anxiety Scale (Hopko, Mahadevan, Bare, & Hunt, 2003) consists of nine items and was used to assess students' mathematics anxiety. All questionnaires were back translated (English-Swedish-English) to ensure that the items measure what they are intended.
Procedure. The study applied a pretest-instruction-posttest design. The pretest (Mathematics skills, interest, self-concept, and anxiety) was administered by one of the researchers during the students' first lesson of the first math course in year 2 in vocational upper secondary education (August 2015). The instruction phase takes place from the middle of August until the middle of October. Both the experimental and the control group will have the same amount of lessons during this time. However, the experimental group will use MinecraftEdu during the course whereas the control group will be provided with "business as usual" instruction. After the course the posttest (Mathematics skills, interest, self-concept, and anxiety) will be administered by one of the researchers.
Statistical analyses. To test the effects of the computer-assisted instruction on students' mathematics skills, interest, self-concept, and anxiety, we will perform a series of multivariate and univariate analyses of covariance. This approach is generally recommended in experimental designs with a pre- and post-test (Maxwell & Delaney, 2004; Rausch et al., 2003).

Findings

At the time of the submission deadline only the pre-test had been administered, so we are not able to report any findings in our extended summary. However, in NORMSA 8 we will present results from both pre- and posttests.

Theoretical and Educational Significance

According to recent meta-analyses computer-assisted instruction has a positive, though small effect on student learning in mathematics (Cheung & Slavin, 2013; Li & Ma, 2011; Rakes et al., 2010). However, to our knowledge, no study has been conducted in Finland in this age-group, nor has this specific computer game (MinecraftEdu) been tested in previous studies. Furthermore, we also incorporate motivational factors (interest, self-concept, & anxiety) that all have strong developmental relations to mathematical skills (Aunola, Leskinen, & Nurmi, 2006; Seaton, Parker, Marsh, Craven, & Yeung, 2014; Vukovic et al., 2013). Following this it is no surprise that for example meta-analytic findings indicate that interventions that target both skills and self-concept are more effective (O´Mara, Marsh, Craven, & Debus, 2006). We want to investigate if this type of instruction can enhance both students' skills and motivational factors in mathematics.

References

Aunola, K., Leskinen, E., & Nurmi, J-E. (2006). Developmental dynamics between mathematical performance, task motivation, and teachers' goals during the transition to primary school. *British Journal of Educational Psychology, 76*(1), 21-40.

Cheung, A. C. K., & Slavin, R. E. (2013). The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. *Educational Research Review, 9*, 88-113.

Frentzel, A. C., Pekrun, R., Dicke, A-L., & Goetz, T. (2012). Beyond quantitative decline: Conceptual shifts in adolescents' development of interest in mathematics. *Developmental Psychology, 48*(4), 1069-1082.

Hopko, D. R., Mahadevan, R., Bare, R. L., & Hunt, M. K. (2003). The abbreviated math anxiety scale (AMAS): Construction, validity, and reliability. *Assessment, 10*(2), 172-182.

Li, Q., & Ma, X. (2011). A meta-analysis of the effects of computer technology on school students' mathematics learning. *Educational Psychology Review, 22*, 215–243.

Marsh, H. W., & O´Neill, R. (1984). Self description questionnaire III: The construct validity of multidimensional self-concept ratings by late adolescents. *Journal of Educational Measurement, 21*(2), 153-174.

O'Mara, A. J., Marsh, H. W., Craven, R. G., & Debus, R. L. (2006). Do self-concept interventions make a difference? A synergistic blend of construct validation and meta-analysis. *Educational Psychologist, 41*(3), 181-206.

Rakes, C. R., Valentine, J. C., McGatha, M. B., & Ronau, R. N. (2010). Methods of instructional improvement in algebra: A systematic review and meta-analysis. *Review of Educational Research, 80*(3), 372–400.

Renninger, K. A., & Su, S. (2012). Interest and its development. In R. Ryan (Ed.), *The Oxford handbook of human motivation*. New York: Oxford University Press (pp.167–187).

Räsänen, P., Linnanmäki, K., Haapamäki, C., & Skagersten, D. (2008). *RMAT - Test av räknefärdighet hos elever i åldern 9-12 år*. Jyväskylä: Niilo Mäki Institut.

Seaton, M., Parker, P., Marsh, H. W., Craven, R. G., & Yeung, A., S. (2014). The reciprocal relations between self-concept, motivation and achievement: Juxtaposing academic self-concept and achievement goal orientations for mathematics success. *Educational Psychology, 34*(1), 49-72.

Vukovic, R. K., Kieffer, M. J., Bailey, S. P., & Harari, R. R. (2013). Mathematics anxiety in young children: Concurrent and longitudinal associations with mathematical performance. *Contemporary Educational Psychology, 38*, 1-10.

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Collaborative research into mathematics teaching and learning in diverse classrooms

Keywords: Teacher development, community of inquiry, mathematics learning

The implementation of the policy of inclusive education in Iceland and the growth of migration has welcomed previously excluded students into schools. As a consequence, teachers are currently faced with new challenges to differentiate teaching. This paper reports on findings from a four years qualitative collaborative inquiry into mathematics teaching and learning with the purpose of deepening our understanding of how teachers meet new cultural and mathematical challenges in their classrooms. The aim was to understand what characterizes the learning processes that emerge through collaborative inquiry where classroom teachers and a teacher educator research their practices together? The focus in this paper is on the teachers' learning and their mutual support in finding ways to assist all children in learning mathematics.

Seven primary school teachers researched their mathematics teaching together with a mathematics teacher educator. The study built on former research on teacher development in mathematics teaching in Iceland that revealed that teachers take a passive role in their mathematics teaching and lack experience in creating meaningful learning environments for all children (Guðjónsdóttir, Kristinsdóttir, & Óskarsdóttir, 2007). On a monthly basis the teachers and the researcher met at workshops where the focus was both on mathematics teaching and learning and teacher reflections on their own teaching. The study was cyclic and experiences from former cycles guided the steps taken in the following cycles. As the teachers refined their teaching spirals of experience emerged and the group learned from former cycles while building new.

The study is a collaborative inquiry into mathematics teaching and learning (Goos, 2004), and the aim is to build a co-learning partnership between teachers and a researcher in promoting classroom inquiry (Jaworski, 2006). In an attempt to make explicit the 'practice' in which teachers and researchers participate when collaborating, Jaworski (2003) suggests shifting from the notion of community of practice to that of 'community of inquiry', where teaching is seen as learning-to-develop-learning. In such a community, teachers and researchers both learn about teaching through inquiring into it.

The workshops (17 in total) were videotaped, and the teachers collected data from their mathematics classes as well as from mutual visits to each other's classrooms. Four interviews with each of the teachers were audiotaped and notes kept from three visits to each of their classrooms. Narrative inquiry was used as an analyzing tool to study the teachers learning in participating in this project. The narrative inquiry is a way of understanding and researching into experience through collaboration between a researcher and participants. The inquiry is collaborative and begins and ends with respect for lived experiences, asking participants to open up their practice and tell their stories (Clandinin, 2013).

The results indicate that the teachers gained confidence in teaching mathematics in diverse classrooms as they participated in workshops and that collaborative research can support teachers in developing their practice when meeting new challenges in their work. When the research project started the teachers emphasized rote learning and memorization and little emphasis was on initiative from the students and discussing different ways of solving problems.

When the project developed the teachers told about their work as their interest in investigative work and discussing with their pupils was growing. Dóra discussed how the children in her group were developing in their mathematics learning:

> They are more willing to discuss their thinking than before and not afraid of doing mistakes. There are though three boys that work differently, have problems with explaining how they solve problems. I need to support them, they hardly ever participate in our discussions, I'm not happy about that.

The focus on their pupils learning shifted gradually from trying to detect their weaknesses to looking for their strengths. Inga told the following story at a workshop:

> I enjoyed this morning's lesson. …We were working on a whale project. The boys got the task to draw the Blue Whale. They went outside with a measuring wheel to visualize how big he is. … Atli started to draw on the pavement, had done the mouth and everything. Then Hilmar said: "This is a bit small mouth for all this whale". Atli just started to draw and did not think that he needed to draw in scale with the total length.

Inga then told us that Hilmar had been labeled as a slow learner and she discussed further how this instance helped her look closer for what the children are capable of doing instead of always searching for what they can not do.

> One is always thinking; do they find a way? … We are more aware of the small steps we are taking. One is more aware of the development.

Pála told us about her experience of working on problems with her pupils that urged them to investigative into the relationships between the operations and with symbolism.

> When we work with such problems, so much happens. Therefor it is important to allow the pupils to deal with such problems and give them the time they need to develop their own thinking about them.

By the end of the four years collaboration the teachers all emphasized that their pupils approached their work from different angles, discussed their work and explained how they understand the mathematics they were dealing with.

The results support other research findings on teacher participation in developmental projects that research with teachers into their own teaching can add to the knowledge base of teaching in schools and teacher development (Cochran-Smith & Lytle, 2009; Norton, 2009). The mutual learning of the participants that developed within the project, needs to be explored further with the goal of gaining more insight into factors that were vital in the developmental process. As Artigue (2009) has emphasised, research with teachers in schools help researchers to take into account factors internal to the field itself, and as learned in this research project, the teachers develop their competence in teaching mathematics.

References:

Artigue, M. (2009). Didactical design in mathematics education. In C. Winsløw (Ed.) *Nordic Research in Mathematics Education. Proceedings of NORMA08* (pp. 7–16). Rotterdam: Sense Publishers.

Clandinin, D. J. (2013). *Engaging in narrative inquiry.* Walnut Creek, CA: Left Coast Press.

Cochran-Smith, M., & Lytle, S. L. (2009). *Inquiry as stance. Practitioner research for the next generation.* New York, NY: Teachers College Press.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258–291.

Guðjónsdóttir, H., Kristinsdóttir, J. V., & Óskarsdóttir, E. (2007). Mathematics for all: Preparing teachers to teach in inclusive classrooms. In L. Østergaard Johansen (Eds.), *Mathematics teaching and inclusion. Proceedings of the 3rd Nordic research conference on special needs education in mathematics* (pp. 123–136). Aalborg: Aalborg University.

Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: Towards a theoretical framework based on co-learning partnerships. *Educational Studies in Mathematics, 54*(2/3), 249–282.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*(2), 187–211.

Norton, L. S. (2009). *Action research in teaching & learning: A practical guide to conducting pedagogical research in universities.* London: Routledge.

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Developing test materials for developmental dyscalculia for Danish pupils in grade 4.

Background

The developmental work follows a political agreement June 2013 between the then government (The Social Democrats, The Danish Social Liberal Party,The Socialist People's Party) and The Left, Denmark's Liberal Party and The Danish People's Party. The agreement deals with improving Danish school children's performance in school subjects (Danish: Aftalen om et fagligt løft af folkeskolen, juni 2013). The agreement from 2013 includes initiatives for general improvements of students' learning and outcome level, especially in mathematics among other chosen subjects. Also the agreement includes specific initiatives for students with dyscalculia (Danish: en målrettet indsats for elever med talblindhed).

The agreement was in 2014 followed by an extensive reformation of the primary and lower secondary school. Also in 2014 the Danish Ministry of Education (DME) released a tender process for a specific developmental project on dyscalculia and after that, a consortium of Danish School of Education, Aarhus University and the University College Zealand got the job offered.

The aim specified by Danish Ministry of Education

The specific developmental project concerns development of a test for dyscalculia for students in Danish Grade 4 and of electronic guidance for follow-up initiatives with the aim of supporting early targeted initiatives in relation to the inclusion of students with dyscalculia in mainstream education.

According to DME the development of a test for dyscalculia shall be based on concepts of dyscalculia, which understand dyscalculia as a learning disability or learning disorder, which can be identified and delimited on research anchored basis of knowledge. As starting point DME pointed at the research overview made by SFI - the Danish national centre for social research ''Talblindhed – en forskningsoversigt' (SFI, 2013).

DME underlined that information, telling that a student is dyscalculic, can provide focus for how to provide relevant focus for attempts towards inclusing learning settings, where students gets targeted training (Danish: målrettet undervisning).
DME also underlined that a standardised dyscalculia test will support municipalities' efforts to give students with dyscalculia an adequate educational offer (Danish: et fyldestgørende undervisningstilbud, ensuring they get possibility to complete education instead of not starting education or instead of drop-out of education. DME wrote, that identification of difficulties as early as possible is a prerequisite for this.

Some existing test on dyscalculia around the world are published by private firms and sold to psychologists, teachers, schools or other institutions. However, DME decided to provide the test from the developmental project free of charge to schools and municipalities from 2018.

Our theoretical framework and project design

Our research literature review includes central publications and results in the following theoretical statements:

We recognise broad research support for dyscalculia as a neurological dysfunction and a developmental disorder.

The concept of *developmental disorder* implies, that dyscalculia may show up in different forms and with different signs from one individual to another individual. Also forms and signs may change through an individual's life, partly as a result of the person's strategy development:

- what an individual is able to is not identical from grade 2, grade 4 or as an adult
- signs may differ among individuals
- in all ages, problems may rise which educational institutions and systems ought to help the individuals to cope with

Based on the above together with DME's aim of providing guidance for supportive actions, we argue that a test – with test questions on paper as well as on screen – as the one and only mean is insufficient to identify dyscalculic students and provide guidance. We argue that teachers' structured interviews and conversations also are to be developed as part of the developmental project into relevant tools for identifying dyscalculic students and for providing guidance.

Especially we will present arguments for including a study in the developmental project of potentially dyscalculic adolescents' and adults' experiences and perceptions of their present challenges with number, calculations and mathematical concepts, as well as of their memories from primary and lower secondary school. We do not expect any simple deterministic relation between an individual in 4. grade and the same individual as an adult. However, adolescents and adults are highly relevant informants. This will be further elaborated in the presentation.

References:

Barrouillet, P., Fayol, M., & Lathulière, E. (1997). Selecting between competitors in multiplication tasks: An explanation of the errors produced by adolescents with learning difficulties. *International Journal of Behavioral Development*, *21*, 253-275.

Butterworth, B. (1999). *The Mathematical Brain*. London: Macmillan.

Butterworth, B. (2005). Developmental dyscalculia. In J.I.D. Campbell (Ed.), *Handbook of Mathematical Cognition.* New York: Psychology Press.

Geary, D.C., & Hoard, M.K. (2001). Numerical and arithmetical deficits in learning-disabled children: Relation to dyscalculia and dyslexia. *Aphasiology*, 15(7), 635-647.

Mazzocco, M.M.; Feigenson, L. & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development*, 82(4), 1224-1237.

Noël, M-P., Grégoire, J., Meert, G. & Seron, X. (2008). The innate schema of natural numbers does not explain historical, cultural, and developmental differences. *Behavioral and Brain Sciences*, 31, 6, 664-665.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., Dehaene, S. & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, 116, 33-41.

SFI (2013). *'Talblindhed – en forskningsoversigt'*. Copenhagen: SFI, 13:34

Streit, G. (2013). *Dyskalkulie/Rechenschwäche/Rechenstörungen.* Deutsche Schule Budapest.

Undervisningsministeriet (2013). *Aftalen om et fagligt løft af folkeskolen* http://www.uvm.dk/~/media/UVM/Filer/Udd/Folke/PDF13/130607%20Aftaleteksten.ashx

Wilson, A. J., & Dehaene, S. (2007). Number sense and developmental dyscalculia. . In D. Coch, G. Dawson & K. Fischer (Eds.), *Human behavior, learning, and the developing brain: Atypical development* (2nd ed.) (pp. 212-237). New York: Guilford Press.

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The arithmetic learning of a low-achieving child from playing a digital game.

Introduction

Children´s understanding and mastery of the part-whole relations of the first numbers 1 to 10 are critical for their further arithmetic development (Anghileri, 2000; Baroody and Tiilikainen, 2003). According to Baroody et al (2009) the learning of arithmetic starts when children are 2 to 4 years old with de development of the intuitive numbers one, two and three. Important for development of arithmetic is also to be competent in compose and decompose numbers (e.g. 2+6=8; 5+4=9). This ability makes it possible for the child to discover patterns and regularities in addition and subtraction, which is the ground for becoming proficient and flexible at mental arithmetic. Children who experience difficulties with arithmetic seem to lack this ability to use simpler arithmetic facts to derive other facts (Gray & Tall, 1994) and instead they become reliant on counting as the only method.

Fingers also play a fundamental role in learning arithmetic. Children use them as tools both when they learn to count, when they answer questions about how many and later on to solve addition and subtraction problems. *Embodiment* is thus an important dimension of mathematics and not the least in learning mathematics early in life (Edwards & Robutti, 2014: p. 2). Neuman (1987) noted that older children with mathematics difficulties often only use counting one by one on their fingers when they shall perform a computation, and argued that a more structured use of fingers displaying part-whole relations would be more productive. According to the theory underpinning the study, the Gibsonian *ecological psychology* (Gibson, 1986) and particular the theory of *perceptual learning* (Gibson and Pick, 2000), perception is the ability to select and "picking up" information when acting in the material world. The theory is used to study if and how the game *Fingu* is supporting the development of arithmetic competence.

Fingu is a game that applies the principles of perceptual learning and is designed to encourage *massive experience* with many and varied tasks, as the theory of perceptual learning prescribe. In the game the player is exposed to two moving sets of objects (e.g. 2 and 4 apples), and is supposed to tell how many objects there are in total (6) by pressing down the corresponding number of fingers on the screen. The fingers must be pressed down roughly at the same time and there are no restriction concerning what fingers are used. There are seven levels in the game with increasing sums and more challenging patterns of objects. To solve this task thus often requires a transformation from the exposed partition of the sum to another partition given by the hands. Thus the

player is stimulated to focus on the parts and the total sum instead of enumerating the sum. Learning to manage the fingers to express sums is in this way an essential part of what Fingu provides. Fingu also affords building up differentied wholes, drawing on the ability to subitize the parts, and develop a conceptual subitizing of the whole.
In pace with the increased digitization of school and preschool digital games are used in mathematics education as a part of other teaching materials. It is not obvious that games benefit children who are low achievers in mathematics. Studies of games designed for learning has shown that there can be problems when children just "learn to play the game" rather than learn the content (Linderoth, 2012). The design of the game aims to contribute to the development of certain competencies which can be contrasted to the player's aim. It is therefore important to study if a game designed to develop arithmetic competence supports this development.

Aims
This case study aims to describe an initial analysis of a child, who is low achieving in arithmetic, and his interplay with a digital game, Fingu. Which strategies can be seen in his playing of the game and how do they develop and what does he learn?

Methodology
During an eight week intervention the children were filmed three times when playing the game Fingu, in the beginning, during the intervention and after the intervention. The children were also tested with pre- and posttests that was verbally distributed.This case was selected from children with low performance on two arithmetic tests, Tema-3 (Ginsburg and Baroody, 2003) and an arithmetic problem solving test. Jacob had low results on pre-test and did not make improvements between pre- and post-test. The initial analysis is based on video 1 and 2.

The case
Jacob is a six-year old boy attending a preschool class. When he played the game during the intervention he developed more effective methods to solve the arithmetic tasks, at least at the first three levels in the game. Jacob played in total 800 tasks during the intervention.
At the levels 4 to 6 Jacob did not use an effective system for counting larger configurations of fruits and he failed to solve the tasks correctly. He did not manage to coordinate the counting words with the pointing to the pieces of fruits.
He almost never used his fingers when he solved the verbal tasks but when he played the game, he was confident in using his fingers. Jacob was attentive to feedback from the game that his response was incorrect and evaluated his reply by studying his fingers to see what was wrong. In the verbal test he had no possibility to do so because he did not get any feedback at all.
In conclusion we can identify two types of strategies that Jacob develops as he plays Fingu. The first is to use counting to find out how many fingers to use in responding to the task. The other is to use some kind of subitizing, perceptual or conceptual (Sarama & Clements, 2009) to directly recognize either the single configuration of a task, the two configurations separately, or the totality of the configurations, all resulting in shorter answering times.

Discussion
In the presentation we will discuss Jacob´s strategies and his learning and what kind of educational significance it can have. We will also discuss the difference between

answering tasks in the verbal test environment versus solving tasks in the game environment.

References

Anghileri, J. (2000). *Teaching Number Sense,* London: YHt Ltd.

Baroody, A. J., & Tiilikainen, S. H. (2003). Two perspectives on addition development. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 75–125). Mahwah, NJ: Erlbaum.

Baroody, A.J., Bajwa, N. P. & Eiland, M. (2009). Why can't Johnny remember the basic facts? *Developmental disabilities research reviews, 15*, 69 – 79.

Edwards, L. D., & Robutti, O. (2014). Embodiment, modalities, and mathematical affordances. In L. D. Edwards, F. Ferrara, & D. Moore-Russo (Eds.). *Emerging perspectives on gesture and embodiment in mathematics.* Charlotte, NC: Information Age Publishing.

Gray, E. M., & Tall, D. O. (1994). Duality, ambiguity, and flexibility: A "proceptual" view of simple arithmetic. *Journal for Research in Mathematics Education, 25,* 116-140.

Neuman, D. (1987). *The origin of arithmetic skills. A phenomenographic approach.* Acta Universitatis Gothoburgensis. Gothenburg.

Gibson, J. J. (1986). *The ecological approach to visual perception*. Hillsdale, NJ: Lawrence Erlbaum.

Gibson, E. J., & Pick, A. (2000) *An Ecological Approach to Perceptual Learning and Development*. Oxford University Press.

Ginsburg, H.P & Baroody, A.J. (2003). TEMA-3: Test of Early Mathematics Ability - Third Edition

Linderoth, J. (2010).Why gamers don't learn more. An ecological approach to games as learning environments, Nordic DiGRA.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research – Learning trajectories for young children.* New York, NY: Routledge.

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CONTENT FLOW in mathematics - A SUPPORT FOR RECOGNITION OF SIMILARITIES ACROSS SITUATIONS

Keywords: content flow, learning situation, inclusion in mathematics, situated knowledge, special educational needs in mathematics

In mathematics education, there has been, and still is, an on-going debate about the assumption that students can easily apply the mathematics learnt in school to their daily life or vice versa, such as work or shopping. Many scholars have problematized this assumption (e.g. Lerman, 1999; Nunes, Schliemann & Carraher, 1993) and argue that this so called transfer does not exist, or at least poor. Instead, they argue that the knowledge is somewhat situated in space, time and activity. If this move between the school context and the regular-life context is hard to do, how about movement between different situations within a school context, such as moving between regular mathematics education and special education in mathematics? In this paper I intend to discuss this issue of situated learning and teaching within the school context for students in special educational needs in mathematics (SEM-students). When the SEM-students move between different teaching situations in mathematics, we cannot make the assumption that the SEM-students can make the transfers we assume them to do in the different situations. How to support students to recognise this so called transfer is the main question in this paper.

Theoretical framing

The issue discussed in this paper is a partial result from a study of inclusion in mathematics from a teacher perspective (Roos, 2015). In this study, two theoretical perspectives was used, a participatory and an inclusive perspective.

To be able to identify how the participation in the mathematics education looked like, a participatory approach was used. This investigation was grounded in a social theory on learning, where learning is considered to be a function of participation (Wenger, 1998). Wengers (1998) social theory on learning is used in many different ways in research (Roos & Palmer, in press). In this particular research only a part of this social theory was used, communities of practice. The notion of transfer does not work within this situated perspective since here knowledge is situated in space, time and activity. Hence, I will use the term similarities instead of transfer, in order to highlight the situated perspective.

An inclusive approach was also used in the investigation, specifically the notions *spatial, social* and *didactical* inclusion by Asp-Onsjö (2006). Spatial inclusion basically refers to how much time a student is spending in the same room as his or her classmates. The social dimension of inclusion concerns the way in which students are participating in the social, interactive play. Didactical inclusion refers the way in which the students engage in the teaching, with the teaching material, the explanations and the content that the teachers may supply for supporting the student's learning. These three terms (spatial, social and didactical inclusion) are used together with communities of practice as an overall frame in developing an explanatory framework.

The study

In order to investigate ways to get students in SEM included in the mathematics in school an ethnographical approach was used in a school context. The data construction was made during a two-year period. A remedial teacher in mathematics with great experience of teaching mathematics to SEM-students was contacted, a choice made in order to get "a best case scenario". Patton (2002) describes this kind of choice as an information-rich case for study in depth. In the overarching study the remedial teacher, the mathematics teachers at the primary school and the principal were interviewed. Also observations were made, both in mathematics classrooms and when the remedial teacher worked with SEM-students. This data served as the basis for identifying the communities of practice at the school, community of mathematics classroom, community of special education needs in mathematics, community of mathematics at Oakdale Primary School and community of student health.

When taking an ethnographic approach, the researcher tries to understand a phenomenon through interpersonal methods. The basis of ethnographic research is social interaction (Aspers, 2007). Categorisation is used for interpretation, which is generated through data analysis (Hammersley & Atkinson, 2007). The data in this research was created by the questions and answers in the interviews and observations.

Findings

To strengthen the teaching of mathematics, the mathematics teachers and remedial teachers in mathematics need to be aware of different ways of supporting the SEM-students. Even more important, they need to be aware of *how* the mathematical content is taught within different teaching situations.

The results show that there are different levels in the teaching of mathematics that need to be considered: both the content level, which representations and tasks are suitable depending on the content, and the student level, which representations and tasks are suitable for this student in this situation. These levels need to be discussed by the mathematics teachers involved in the different teaching situations. The teachers cannot assume that the SEM-students recognise the similarities in different situations. Hence, there is a need in the teaching of mathematics to support the students recognise the similarities in order to achieve learning situations.

There is three aspects of teaching support visible in the data, *prepare, immerse and repeat* I call *content flow*. All three can be applied, but depending on the student(s), the mathematical content and the situation, only one or two aspect(s) could be applied. Hence, the *content flow* is used in the teaching of mathematics between the communities with help of tasks and representations.

The result shows that representations are important in mathematics education and very important when talking about special educational needs in mathematics. The teaching in mathematics need to make the students aware of, and able to handle, different representations, and the teacher needs to have knowledge of the use of different representations in relation to a mathematical content. Consequently, representations in mathematics need to be considered as a part of the teaching and learning of SEM-students.

References

Asp-Onsjö, L. (2006). *Åtgärdsprogram dokument eller verktyg? En fallstudie i en kommun*: Diss. Gothenborg : University of Gothenburg, 2006. Gothenborg.

Aspers, P. (2007). *Etnografiska metoder: att förstå och förklara samtiden.* Malmö: Liber.

Flyvbjerg, B. (2006). Five misunderstandings About Case-Study Research. *Qualitative Inquiry, 12* (2), 219-245.

Hammersley, M. & Atkinson, P. (2007). *Ethnography. Principles in practice.* (3rd ed.). London: Routledge.

Lerman, S. (1999). "Culturally situated knowledge and the problem of transfer in the learning of mathematics." *Learning mathematics: From hierarchies to networks:* 93-107.

Nunes, T., Schliemann, A.D. & Carraher, D.W. (1993). *Street mathematics and school mathematics.* Cambridge, UK: Cambridge University Press.

Patton, M. Q. (2002). *Qualitative research and evaluation methods* (3rd ed.). London: Sage.

Roos, H. (2015). Inclusion in mathematics in primary school: what can it be? Licentiate thesis. Växjö: Linnéuniversitetet, 2015. Växjö.

Roos, H. & Palmer, H. (in press) Paper presented at the Congress of the European Society for Research in Mathematics Education in Prag, 2015.

Wenger, E. (1998). *Communities of practice. Learning, Meaning and Identity.* Cambridge: Cambridge University Press.

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Exploring the affective domain in the teaching of mathematics – A qualitative study on students' perspectives on math in the Danish public school (primary education)

How can we access primary school students' perspectives on mathematics?

The purpose of the TMTM project is to bring forth knowledge on how marginalized students - low as well as high achievers - think mathematically, as a point of departure to initiate more effective teaching of the two groups of students. The substudy is delimited to obtain knowledge on TMTM students' experience of the intervention program and the teaching of mathematics in general settings.

The substudy proceeds with two interwoven purposes:

1) The first has to do with the development of a qualitative research method that creates a constructive basis for helping primary school children verbalize their perspectives.
2) The second has to do with producing knowledge on how primary school students actually relate to mathematics. The focus is on the students' attitudes in relation to the teaching of mathematics.

Affect encompasses complex structures

An important early contribution in conceptualizing the field of affect was identifying beliefs, attitudes, and emotions as elements of the affective domain (McLeod, 1992). As Goldin, Rôsken and Törner (2009) write, beliefs are no longer a hidden variable in mathematical teaching and learning processes, but as pointed out by Törner (2013), the notion of 'belief' is a fuzzy construct. At the same time it is difficult methodologically to design and carry out reliable empirical studies of affect (Debellis & Goldin, 2006). There are ongoing controversies about identifying and characterizing affective aspects of mathematics education.

Recent research on affect takes into account a radical critique:

> "…the limits of a normative approach, i.e. the attempt to explain behaviour through measurements or general rules based on a cause-effect scheme. The awareness of the high complexity of human behaviour gradually led to the affirmation of an alternative paradigm: the interpretive one, aimed at understanding – rather than explaining through universal laws – an individual's actions." (Zan, 2013, p. 52)

Qualitative approaches integrate theories of problem solving, affect, and motivation or learning and teaching, and seek insights through statements and behavior of students (Goldin, Rôsken & Törner, 2009). But according to McDonough and Sullivan (2014) few studies of views on mathematics have included younger participants. Furthermore, McDonough and Sullivan point to the identification of prompts as a challenge for researchers – prompts to which young children can respond easily, and which have the potential to provide meaningful insights into their beliefs.

Beliefs can be associated with both cognitive and affective domains. While acknowledging differing approaches and definitions we see affect as encompassing complex structures of emotional responses and feelings, attitudes, beliefs, and values as these interact with cognition; hence, this case study explores relations between emotions, visions of mathematics, and perceived competence (Zan & Di Martino, 2007; Zan, 2013).

International studies show connections between achievement and attitudes. TIMSS 2011 demonstrated that in almost all countries "students who reported not liking learning mathematics had the lowest average mathematics achievement (Mullis, Martin, Foy, & Arora, 2012, p. 328). In PISA 2012 it became apparent that students who were motivated to learn mathematics and who had a positive image of themselves as mathematics learners "perform better in the mathematics assessment" (OECD, 2013, p. 36). In continuation of these findings, the case study focuses on students' attitudes and the analytical resources draw support from the "Three-dimensional Model for Attitude" (Di Martino & Zan, 2010). Based on this model, fig. 1 underlines students' attitudes as situated in a social context: mathematical teaching.

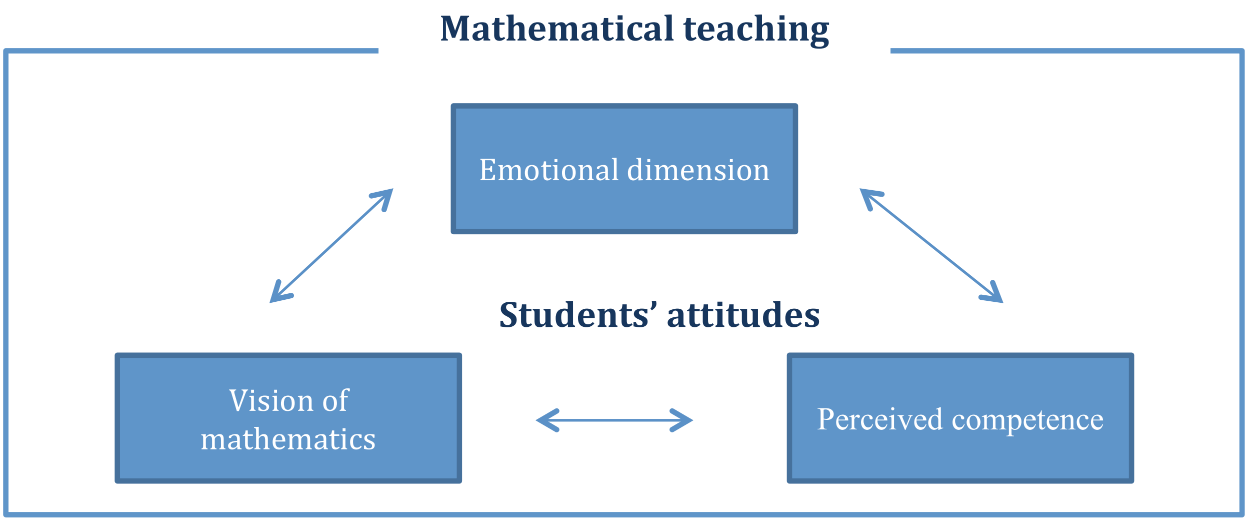

Fig.1. Relations in math students' attitudes

Conducting photo elicitation interviews – methodological constructs

Locating the research within an interpretivist tradition of constructivism (Ferguson, 2009), using qualitative methods and a case study design, this investigation explores patterns in 20 student interviews in ten mathematics classrooms at five schools. The students are low as well as high achievers in 3rd grade.

Attempting to capture 3rd grade students' voices, the interviews are shaped as conversations about self-produced images of math, specifically photographs produced by the students themselves as well as the researcher. The photos are taken in the classroom setting of general mathematics teaching. When conducting photo elicitation interviews, researchers introduce photographs into the interview context, and photographs of the child's experiences serve as the basis for a child-directed interview (Clark, 1999). But little has been written about the use of photographs in interviews with children (Epstein et al., 2006). In most research the photos are taken either by the researcher or by the participants, but in this study photographs are produced by both.

The photos represent topics that lead to semi-structured interviews exploring five overall research questions:

- What do 3rd grade students think is typical of math as a subject?
- What do 3rd grade students think is typical of a lesson in math?
- What do 3rd grade students think matters in relation to their participation?
- What do 3rd grade students think the 'perfect' math class should look like, if they were to design it?
- What do 3rd grade students experience as different in TMTM teaching compared to the teaching of mathematics in general?

Anticipated educational significance of the research

As part of the overall study - the TMTM project, our primary objective concerns on the one hand, marginalized groups and on the other, bringing forth this knowledge in relation to a broader perspective - students *not* in math difficulties.
Specifically, the primary aim is to extend and enhance the practice of using screening tests involved in identifying and assessing further interventions, by making didactical use of (methodological) findings on communicative practices and students' perspectives from interviews. In addition, this study also attempts to heighten teachers' general awareness of the affective aspects of students' participation strategies in everyday mathematics, and deliver methods on how to gain further insight.
Our framework ultimately derives from an international discourse on human rights, moving children's perspectives closer to the center of the educational agenda (Office of the United Nations High Commissioner for Human Rights, 1990, article 12). This links the use of affective insights in mathematical teaching to an overall political orientation toward making way for children's voices and participation.

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References

Clark, C., D. (1999) The Autodriven interview: A photographic viewfinder into children's experience, *Visual Sociology*, 14:1, 39-50, DOI: 10.1080/14725869908583801

DeBellis, Valerie A., & Goldin, Gerald A. (2006). Affect and Meta-Affect in Mathematical Problem Solving: A Representational Perspective. *Educational Studies in Mathematics*, 63(2), 131-147

Di Martino, P., Zan, R. (2010). 'Me and maths': towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teacher Education*, 13 (1), 27–48.

Epstein, I., Stevens, B., McKeever, P. & Baruchel, S. (2006). Photo Elicitation Interview (PEI): Using Photos to Elicit Children's Perspectives. *International Journal of Qualitative Methods* 5 (3) 1-10.

Ferguson, D.L. (2009). Introduction: Honoring and celebrating diversity in educational research (p. 9-18). In: B. Sundmark (ed), *EDUCARE 2009: 4 Att infånga praxis - kvalitativa metoder i (special)pedagogisk forskning i Norden*. Malmö högskola: Malmô.

Goldin, G., Rôsken, B. & Tôrner, G. (2009). Beliefs – No longer a hidden variable in mathematical teaching and learning procceses. In J. Maasz & W. Schloeglmann, (Eds*.) Beliefs and attitudes in mathematics education. New research results.* Rotterdam: Sense.

McLeod, D. M. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: Macmillan.

McDonough, A. & Sullivan, P. (2014) Seeking insights into young children's beliefs about mathematics and learning. *Educational Studies in Mathematics*, Retrieved 24.06.2014 from http://link.springer.com/article/10.1007/s10649-014-9565-z#

Mullis, I.V.S., Martin, M.O., Foy, P., & Arora, A. (2012). *Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College*.
http://timss.bc.edu/timss2011/international-results-mathematics.html
OECD. (2013). *PISA 2012 results: Ready to learn: Students' engagement, drive and self-beliefs (Volume III)[Preliminary Version]* http://www.oecd.org/pisa/keyfindings/PISA2012-Vol3-Chap2.pdf

Törner, G. (2013). Solid Findings in Mathematics Education: Living with beliefs and orientations – underestimated, nevertheless omnipresent, factors for mathematics teaching and learning. *EMS Newsletter*, March 2013, pp. 42–44.

Zan, R. (2013). Solid Findings on Students' Attitudes to Mathematics. *EMS Newsletter*, September 2013, 51-53.

Zan, R. & Di Martino, P. (2007). Attitude toward mathematics: overcoming the positive/negative dichotomy. *The Montana Mathematics Enthusiast*, Monograph 3, 157-168.

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Tal och resonemang om representationer i förskoleklassen

Förskolebarns matematiska kunskaper vid skolstarten har starka samband med senare generell skolframgång och med prestationer i matematik i grundskolan (Duncan et al., 2007). Särskilt betydelsefull tycks den utveckling som sker mellan cirka 4,5 och 7 års ålder vara (Watts et al., 2014). Internationella studier har visat att barn som börjar skolan med ett alltför begränsat kunnande i matematik riskerar att utveckla matematiksvårigheter i grundskolan i en nedåtgående spiral (Jordan, Kaplan, Locuniak & Ramineni, 2007; Morgan, Farkas, & Wu, 2009). Forskningens intresse för interventioner före skolstarten och i tidiga skolår har ökat under senare år. Exempel på interventioner som har utvärderats vetenskapligt med goda resultat är *Building Blocks* (Clements et l., 2013) och *Number Worlds* (Griffin, 2007). Förskoleklassen har en unik ställning i det svenska utbildningssystemet som bryggan mellan det informella lärande som oftast dominerar i förskolan, och det mer formella lärande som tar vid i skolan. Med utgångspunkt i en kunskapsöversikt identifierades fem matematiska teman och resonemang om representationer som kärnan i undervisning och lärande, med fokus på barns utveckling av taluppfattning före den formella skolstarten.

Ett matematiskt pedagogiskt program, med strukturerad explicit undervisning och fokus på tal och barns och lärares kollektiva resonemang om representationer, har prövats ut i samverkan mellan forskare och verksamma förskoleklasslärare (Sterner & Helenius, 2015). En metaanalys (Gersten et al.,) indikerar att strukturerad explicit undervisning är särskilt gynnsam för elever i riskzonen för att utveckla matematiksvårigheter. Avsikten med det matematiska pedagogiska programmet var att utgöra ett stöd för lärarna i förskoleklass och undervisningen om tal och tals användning. Programmet har prövats ut med iterativ metod i fyra faser och för varje fas har en ny grupp förskoleklasslärare rekryterats. Sammantaget medverkade 26 förskoleklasslärare i den iterativa utprövningen. De matematiska aktiviteterna som ingår i programmet bygger på forskning och teori om barns utveckling av taluppfattning och om hur undervisningen kan främja denna utveckling (t ex Aunio, Hautamäki & Van Luit, 2005; Clements & Sarama, 2007; Clements et al., 2013; Dyson, Jordan & Glutting, 2011; Griffin, 2007; Nunes et al., 2007). Syftet med den iterativa utprövningen av aktiviteterna var att förfina tre teoretiska principer som utgör modellens ramverk, samtidigt som också det konkreta programmet kunde förbättras. Metodologiskt inordnas denna studie under pedagogisk designforskning (McKenney & Reeves, 2012).

Med utgångspunkt i ovan nämnda program genomfördes en tio veckors matematisk randomiserad intervention med barn i förskoleklass. Studiens övergripande syfte var att på gruppnivå studera effekter av interventionen i förskoleklass på barnens senare matematikkunskaper i årskurs 1. I studien deltog tolv förskoleklasser, sex klasser i interventionsgrupp och sex klasser i kontrollgrupp. I ett första steg lottades de tolv klasserna slumpmässigt in i en interventionsgrupp eller en kontrollgrupp. I ett andra steg lottades barnen inom varje förskoleklass, så att alla barn deltog i undervisningen men bara hälften testades och deltog därmed i studien (62 barn i respektive grupp). Experimentgruppen erhöll 30 minuter daglig matematikundervisning under tio veckor. Utöver den matematiska interventionen erhöll experimentgruppen ingen ytterligare matematikundervisning. Kontrollgruppen erhöll 30 minuter daglig undervisning i fonologisk medvetenhet under tio veckor. Undervisningsmaterialet som användes är teoretiskt välförankrat (Lundberg, Frost & Petersen, 1988) och är vanligt förekommande i förskoleklassen. Kontrollgruppen erhöll ingen ytterligare undervisning i fonologisk medvetenhet utöver den fonologiska interventionen och följde för övrigt ordinarie matematikundervisning. Effekten av den matematiska interventionen i experimentgruppen jämfördes alltså med effekten av den ordinarie matematikundervisning som erbjöds inom förskoleklassen. Undervisningen genomfördes i båda grupperna av barnens egna lärare. Lärarna i experimentgruppen deltog i kompetensutveckling i matematik vid sammanlagt sju seminarier/föreläsningar. Samtliga lärare hade tidigare deltagit i kompetensutveckling med fokus på fonologisk medvetenhet i enlighet med det program som användes i kontrollgruppen (Lundberg, 2007). Förtest, eftertest och uppföljningstest har genomförts. På förtestet hade kontrollgruppen signifikant bättre resultat på matematiktestet i jämförelse med experimentgruppen. Kontrollgruppen hade även signifikant bättre resultat på test av visuellt arbetsminne. Däremot fanns inga skillnader mellan grupperna på test av verbalt arbetsminne. Resultaten visade en signifikant effekt av interventionen på matematik vid eftertestet till interventionsgruppens fördel. Det fanns också en bestående effekt av interventionen på barnens matematikkunnande nio månader senare då de gick i årskurs 1. Studien bidrar sammantaget till att demonstrera att strukturerad och explicit undervisning i förskoleklass med fokus på tal, resonemang och representationer har signifikant positiv effekt på utvecklingen av barnens matematiska kunnande (Sterner, 2015; Sterner, Wolff & Helenius, manus).

Referenser

Aunio, P., Hautamäki, J. & Van Luit, J. E. H. (2005). Mathematical-thinking intervention programmes for preschool children with normal and low number sense. *European Journal of Special Needs Education, 20*, 131-146. DOI:10.1080/08856250500055578

Clements, D. H. & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the Building Blocks project. *Journal for Research in Mathematics Education, 38,* 136-163.

Clements, D. H., Sarama, J., Wolfe, B. B. & Spitler, M. E. (2013). Longitudinal evaluation of a scale-up model for teaching mathematics with trajectories and technologies: persistence of effects in the third year. *American Educational Research Journal, 50*, 812-850.

Duncan, G. J., Claessens, A., Huston, A. C., Pagani, L. S., Engel, M. et al. (2007). School readiness and later achievement. *Developmental Psychology, 43*, 1428-1446.

Dyson, N. I., Jordan, N. C. & Glutting, J. (2011). A number sense intervention for low-income kindergartners at risk for mathematics difficulties. *Journal of Learning Disabilities 46,* 166-181.

Jordan, N. C., Kaplan, D., Locuniak, M. N. & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research and Practice*, 22, 36-46.

Griffin, S. (2007). Early intervention for children at risk of developing mathematical learning disabilities. In D. B. Berch & M. M.M. Mazzocco (Eds.), *Why is math so hard for some children?* (pp. 373-396). Baltimore: Paul H. Brookes Publishing Co.

Lundberg, I., Frost, J. & Petersen, O-P. (1988). Effects of an extensive program for stimulating phonological awareness in preschool children. *Reading Research Quarterly, 23,* 263-284.

Lundberg, I. (2007). *Bornholmsmodellen: vägen till läsning: språklekar i förskoleklass*. Stockholm: Natur och Kultur.

McKenney, S. & Reeves, T. C. (2012). *Conducting educational design research*. New York: Routledge.

Morgan, P.L., Farkas, G. & Wu, Q. (2009). Five-year growth trajectories of kindergarten Children with learning difficulties in mathematics. *Journal of Learning Disabilities, 42*, 306-321.

Nunes, T., Bryant, P., Evans, D. Bell, D. Gardner, S. et al. (2007). The contribution of logical reasoning to the learning of mathematics in primary school. *British Journal of Developmental Psychology, 25*, 147-166.

Sterner, G., Helenius, O. & Wallby, K. (2014). *Tänka, resonera och räkna i förskoleklassen*. Göteborg: Nationellt Centrum för Matematikutbildning (NCM).

Sterner, G., Wolff, U. & Helenius, O. *Reasoning about representations: effects of an early math intervention.* Manuscript submitted for publication.

Sterner, G. & Helenius, O. (2015). Number by reasoning and representations – the design and theory of an intervention program for preschool class in Sweden. In O. Helenius, A. Engström, T. Meaney, P. Nilsson, E. Norén et al. (Eds), *Development of mathematics teaching: design, scale, effects. Proceedings from MADIF9:* The ninth Swedish mathematics education research seminar, Umeå, February 4-5, 2014. Linköping: SMDF.

Watts, T. W., Duncan, G. J., Siegler, R. S. & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher, 43*, 352-360.

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Developmental trajectories of strategies in arithmetic – the role of decomposition

Introduction

Mental strategies in arithmetic early in school has shown to be valid predictors of later mathematical achievements and difficulties (Gersten et al., 2005; Ostad, 1997). Strategy use depicts underlying factors such as number sense, number knowledge and cognitive development, which are important in developing mathematic proficiency and competence (Butterworth, 2005; Geary, 2013; Vanbinst et al., 2012). Development of strategy use from counting strategies to direct retrieval is described by many studies (Ashcraft, 1982; Carr & Alexeev, 2011) and a high frequency of counting is predictive of possible difficulties (Ostad, 1997).

Strategy choice is constrained by knowledge necessary to execute the strategy (Laski et al., 2014). Counting as strategy is a linear or uni-dimensional thinking of numbers and thus an inflexible way of thinking in addition problems. Decomposition demands a more comprehensive, spatial understanding of relations between numbers and quantity and the ability to partition quantities in multiple ways (Fennema et al., 1998; Geary et al., 2013; Laski et al., 2014).The capacity to decompose numbers is an important part of the later developing of multiplicative thinking (Clark & kami, 1996).

Developmental trajectories of strategy use provide insight into when students in general are capable of learning more complex arithmetic like multiplication and division. Important knowledge when planning teaching in general but especially in relation to early intervention and targeted teaching for students in or at risk of developing difficulties in mathematics.

Albeit the importance of decomposition has long been acknowledged, fact retrieval is often considered the desired outcome (Cowan, 2003) and many studies do not differentiate between counting and decomposition, but distinguish between direct retrieval and 'everything else' (Bailey et al., 2012; Bartelet et al., 2014). The goal of this study is therefor to describe the developmental trajectories of strategy use in mental addition in a Danish school context with emphasis on decomposition.

The results presented here are a part of a study in progress on relations of classroom teaching, teacher knowledge and student development and will be discussed in relation to early intervention and targeted teaching for children with difficulties in mathematics.

Methods

The data consist of 202 tests rounds on 123 students (58 tested once, 44 twice, 21 thrice in different grades) in 1st to 4th grade (54, 62, 55 and 38) from a Danish public school.

Strategy use was monitored in one-to-one assessment interviews by presenting the student with flashcards with the 36 problems with numbers 2-9 (Sunde & Pind, 2014). Strategy categories were: 1) 'Error': gives up/miscalculates, 2) 'Counting': all varieties of counting procedures, 3) 'Direct retrieval': sum is automatized and 4) 'Decomposition': addends are decomposed and automatized sums are used to calculated the answer (e.g. 4+5 = 4+4+1 or 5+5-1).

Variation in frequency by which a given solving strategy was used (e.g. counting as opposed to all other strategies) as a function of school age (number of years the student has attended formal school) and gender was analysed with Generalised Linear Mixed Models (GLIMMIX procedure in SAS 9.4) with a logit link function and binomial error distribution. Student identity, test round nested within student ID and problem identity were stated as random effects.

Results

The frequency by which all four strategy categories were used varied as function of school age in significant interaction with gender (Figure 1). More advanced strategies (decomposition and direct retrieval) were increasingly used with increasing school age, with boys heading in front of girls: in 1st grade girls primarily counted whereas boys used direct retrieval or decomposition. By 4th grade, girls' strategy use nearly equalled that of the boys.

Discussion

The increased use of advanced strategies with increasing school age was expected, the substantial differences between genders at early school age in use of decomposition and counting was not. This could have implications for early intervention and targeted teaching as discussed below.

Reports on gender differences in use and development of strategies are very divers, ranging from none (Lachance & Mazzocco, 2006) to significant differences (Bailey et al., 2012; Carr et al., 2008; Fennema et al., 1998). Gender differences have been explained by many factors (Byrnes et al., 1999; Chang et al., 2011; Geary, 1996). The reasons for the gender differences in the present study are still unknown.

As also reported by others (e.g. Bailey et al., 2012; Fennema et al., 1998), boys were more likely to use direct retrieval than girls. However, I also found substantial differences in the use of counting and decomposition. Carr and Alexeev (2011) found that developmental trajectories for complex arithmetic and the strategy use by the beginning of 2nd grade is more predictive of later mathematical proficiency than the later strategy use. Furthermore Bailey et al. (2012) found that preference for and skill at using a specific strategy was related in a feedback loop: early preference predicts later skill which predicts later preference and so forth. This has implications for early intervention to prevent later difficulties. When some students prefer counting as strategy for solving basic arithmetic, they get less practice in working flexible with numbers and thus have fewer opportunities to develop the more complex understanding necessary in advanced arithmetic like multiplication and division (Gersten et al., 2005). The longer they use inadequate strategies with success (getting the right answer) the greater the effort to change strategy. Thus the students preferring counting will probably be more prone to use this strategy when introduced to e.g. multiplication.

In this study, girls started off by primarily using counting and it is not until around 4th grade their strategy use equals that of the boys. However, this does not necessarily imply that their thinking strategies have converged. It could merely be that strategy use in simple addition is not an appropriate diagnostic in the later grades. This could be verified by testing a more complex arithmetic situation like multiplication. In a study on children in grades 1-5 Clark and Kamii (1996) found that even though 45% of 2nd graders were capable of multiplicative thinking, 18 % of the children in 4th grade only used additive thinking.

In Danish schools multiplication is generally introduced in 2nd grade and division in 3rd. At 2nd grade many of the students largely used counting when solving simple addition. These students may possibly have to use more effort to develop and understand multiplication than their peers.

The current analysis did not provide detailed information on the developmental pathways from counting to decomposition and direct retrieval. The next step will be to investigate these pathways on the individual and group level and established the relations to the development of multiplicative thinking.

Conclusions and perspectives

The developmental pattern of strategy use reported here implicates that decomposition play a role in the developmental trajectories. I suggest that spatial understanding of numbers should be considered in planning early intervention because multi-dimensional understanding of numbers with emphasis on quantitative patterns and partitioning of quantities is important for the further development in arithmetic competence and multiplicative thinking.

Analysis of development in strategy use for specific problems, e.g. near ties, and analysis of the specific developmental trajectories of students provide more insight in the pathways regarding strategies in arithmetic influencing difficulties in mathematics. This may be important knowledge when planning targeted teaching for students with difficulties in mathematics.

References

Ashcraft, M. H. (1982). The development of mental arithmetic: A chronometric approach. *Developmental Review, 2*(3), 213-236.

Bailey, D. H., Littlefield, A., & Geary, D. C. (2012). The codevelopment of skill at and preference for use of retrieval-based processes for solving addition problems: Individual and sex differences from first to sixth grades. *Journal of Experimental Child Psychology, 113*(1), 78-92.

Bartelet, D., Vaessen, A., Blomert, L., & Ansari, D. (2014). What basic number processing measures in kindergarten explain unique variability in first-grade arithmetic proficiency? *Journal of Experimental Child Psychology, 117*, 12-28.

Byrnes, J. P., Miller, D. C., & Schafer, W. D. (1999). Gender differences in risk taking: A meta-analysis. *Psychological bulletin, 125*(3), 367.

Butterworth, B. (2005). The development of arithmetical abilities. *J.Child Psychol.Psychiatry Allied Discipl. 46*(1), 3-18.

Carr, M., Steiner, H. H., Kyser, B., & Biddlecomb, B. (2008). A comparison of predictors of early emerging gender differences in mathematics competency. *Learning and Individual Differences, 18*(1), 61-75.

Carr, M., & Alexeev, N. (2011). Fluency, accuracy, and gender predict developmental trajectories of arithmetic strategies. *Journal of Educational Psychology, 103*(3), 617-631.

Chang, A., Sandhofer, C. M., & Brown, C. S. (2011). Gender biases in early number exposure to preschool-aged children. *Journal of Language & Social Psychology, 30*(4), 440-450.

Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1-5. *Journal for Research in Mathematics Education*, 41-51.

Cowan, R. (2003). Does it all add up? changes in children's knowledge of addition combinations, strategies, and principles. In: Baroody, A.J, Dowker, A. *The Development of Arithmetic Concepts and Skill: Constructing Adaptive Expertise*, 35-74.

Fennema, E., Carpenter, T. P., Jacobs, V. R., Franke, M. L., & Levi, L. W. (1998). A longitudinal study of gender differences in young children's mathematical thinking. *Educational Researcher,* 6-11.

Geary, D. C. (1996). Sexual selection and sex differences in mathematical abilities. *Behavioral and Brain Sciences, 19*(02), 229-247.

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PloS One, 8*(1), e54651.

Geary, D. C. (2013). Early foundations for mathematics learning and their relations to learning disabilities. *Curr.Dir.Psychol.Sci., 22*(1), 23-27.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of learning disabilities, 38*(4), 293-304.

Lachance, J. A., & Mazzocco, M. M. (2006). A longitudinal analysis of sex differences in math and spatial skills in primary school age children. *Learning and Individual Differences, 16*(3), 195-216.

Laski, E. V., Ermakova, A., & Vasilyeva, M. (2014). Early use of decomposition for addition and its relation to base-10 knowledge. *Journal of Applied Developmental Psychology, 35*(5), 444-454.

Ostad, S. A. (1997). Developmental differences in addition strategies: A comparison of mathematically disabled and mathematically normal children. *British Journal of Educational Psychology, 67*(3), 345-357.

Sunde, P. B., & Pind, P. (2014). *RoS - test* (1. udgave ed.). Skødstrup: Pind og Bjerre.

Vanbinst, K., Ghesquiere, P., & De Smedt, B. (2012). Numerical magnitude representations and individual differences in children's arithmetic strategy use. *Mind, Brain, and Education, 6*(3), 129-136.

Figures

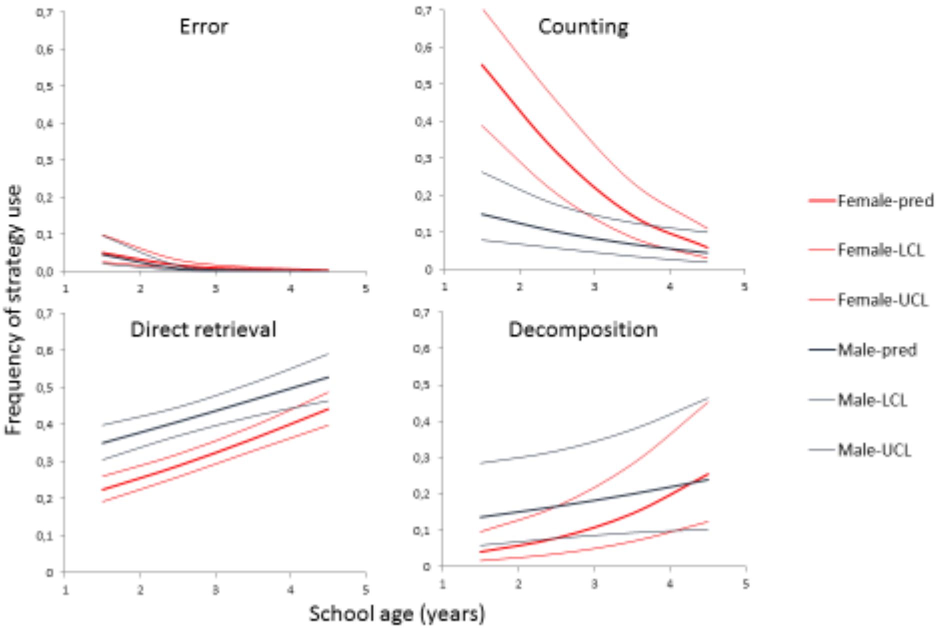

Figure 1: Frequencies of use of the different strategy categories: Error, counting, direct retrieval and decomposition, for girls and boys separately. Pred: predicted value. LCL: 95% lower confidence level. UCL: 95% upper confidence level. School age: number of years the student has attended school.

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Early math intervention for marginalized students – A mixed method substudy of high achieving students

The purpose of the project: How do we deal with the teaching of high achieving students in an early intervention?

The aim of the TMTM 2014 project is to bring forth knowledge of how marginalized students, low as well as high achievers, response to an early intervention and to gain insight into the mathematical thinking and reasoning of these 2th grade (8 years old) students.

The presentation is based on a sub study whose aim is to:

1. Gather information on the teachers' characteristics of the high achieving students.
2. To evaluate their description/characteristics by a comparison with observation of the students participating in the interventions.
3. Examine to what extend this characteristic can be useful for educational initiatives

The view of Mathematics, difficulties and special needs

In the TMTM2014 project we have been inspired by the Danish scholar Ole Skovsmose and the Norwegian scholar Stieg Mellin-Olsen. The paradigm on learning mathematics as landscapes of investigation suggested by Skovsmose (2001), as opposed to the exercise paradigm, has inspired us. The metaphor of a travel suggested by Mellin-Olsen (1991) in teachers' thinking about instruction as a common teacher-student journey inspired us as well. Since 2003 (Böttger, Kvist-Andersen, Lindenskov & Weng, 2004) we have been involved in demonstrating mathematics learning as a journey in which many routes can be appropriate for the teachers and students involved.

This implies a new view on students in difficulties learning mathematics, condensed in the construct 'math holes'. In this metaphor the difficulties are represented by the 'hole' and the surroundings of the hole is seen as a landscape that give the student alternative opportunities to move on. To continue the metaphor, the teacher can invite the struggling student to move to another type of landscape, to fill up the hole or to bridge the cap (Böttger, Kvist-Andersen, Lindenskov & Weng, 2004).

The purpose of the presentation are the high achieving students and their teachers. There may not be a tradition to involve high achievers/gifted students to the group of students with special needs (The Salamanca Declaration 1994), but according to The Common Guidelines issued by Ministery of Education in Denmark, students with special needs include both low and high achievers (Undervisningsministeriet 2009). There is some evidence that gifted students need special support to continue their leaning (Engström 2007). Teachers are not always conscious about the fact that high achievers are included in this group of student with special needs in Denmark or that they need to pay some extra attention to them.

Results from PISA 2012 show that the level of low achievers in Mathematics in Denmark had not change significantly from PISA 2003, and that the proportion of high achievers has significantly decreased compared to PISA 2003 (Undervisningsministeriet 2013) (Lindenskov & Jankvist, 2003)

The agreement text on elementary school points out that the number of high achievers are relatively small (Undervisningsministeriet 2013) and the common objectives suggests special tasks of challenge for high achieving students with special needs in order to improve their competences (Undervisningsministeriet 2014).

Method

In autumn 2014, 281 2th grade students (8 years old) received an individual special teaching, *early intervention*, based on the identical program logic model and materials for both two groups. The material was 'Matematikvanskeligheder – Tidlig intervention' (Lindenskov og Weng 2013). This intervention was implemented by 82 mathematics teachers placed in 41 schools in 31 different Danish municipalities. Every 281 Grade 2 students received an individual special math teaching 20 minutes per day, 4 days a week for 12 weeks. The marginal groups, one for the 20 percent high achieving students and one for the 20 percent low achieving students, were determined by a pretest. The participating students in the intervention programe were randomly selected from these two groups.

In this substudy we focus on the high achieving students who recieved individual teaching.

The same teaching material was used for both the high and low achieving students, but the teachers had the opportunity to organize their teaching according to the need of the individual student. The intervention was initiated by a screening test and an interview to decide where to start their teaching according to the needs of the student, but the planning of the intervention was intented to be made in a formative manner collaboratively between the student and the teacher during the intervention. The teaching material focuses on mathematics as problem solving and see the dialogue between the student and the teacher as a way of cooperation that the student (and the teacher) can benefit from.

3-4 sessions of each of the 281 intervention students were video recorded and these observations are used in our analysis of the teachers' characteristic of the high achieving students. Specifications of the mathematical activities in the intervention selected by the teacher provide us with information of how the teacher has planned the intervention.

A questionnaire survey was conducted to collect information on the teachers' views on the interventions, the students they had been teaching, and subsequent in-depth interviews with 24 teachers, were made in order to get their elaboration of their answers from the survey. The teachers were asked several questions on their view of the intervention and the cooperation with the students and other people involved in the project, but we are interested in their answers to these questions:

Would you have chosen this student as high achiever yourself?
What characterizes a high achieving student?
Was the teaching material useful for a formative evaluation/appropriate for teaching?

Results

In recent years, there has been an increasing focus on high achieving students in mathematics, but teachers still ask for support to deal with the educational challenges these students give them. This claim is confirmed by the result of the survey in which most teachers express uncertainty about their description of the high achieving students. The response rate at 96% in the questionnaire survey suggests a great commitment by the teachers in the project, but the teachers also expressed a great deal of frustration of not being able to deal with this group of students with special needs. The in-depth interviews support this conclusion, but also reveal that several teachers base their characterization of the students on presumptions that are not supported by the video recordings of the interventions.

Although some few developmental projects on high achievers have been taken place in Denmark, it is a newly established political agenda in Denmark to also put special emphasis on high achieving students, so schools teachers do not have much experience in this issue.

References

Böttger, H., Kvist-Andersen, G. Lindenskov, L. & Weng, P. (2004). Regnehuller. I:

Engström, A., (ed.) *Democracy and Participation A Challenge for Special Needs Education in Mathematics. Proceedings of the 2nd Nordic Research Conference on Special Needs Education in Mathematics(p. 121 - 135).* Örebro University.

Engström, A. (2000). Specialpedagogikk för 2000-talet. Nä*mnaren*. nr. 1/2000

Engström, A. (2008). De mest begåvade barnen är sämst på att lära nytt. DN Debatt 2007: http://www.dn.se/debatt/de-mest-begavade-barnen-ar-samst-pa-att-lara-nytt/

Gervasoni, A. & Lindenskov, L. (2011). Students with 'Special Rights' for mathematics education. In B. Atweh, W. Graven, & P. Secada, (Eds). *Mapping Equity and Quality in Mathematics Education,* (pp.307-323). Netherlands: Springer

Kyriacou, C. & Goulding, M (Coordinators). (2004). *A Systematic Review of the impact of Daily Mathematics Lesson in enhancing Pupil Confidence and Competence in early Mathematics.* EPPI-Review.

Lindenskov, L. & Weng, P. (2009). "Math-holes" – theory and empirical data. A pilot study in school year 1-3. In: Linnanmäki, K. (Ed). Different Learners – Different Math?

The 4th Nordic Research Conference on Special Needs Education in Mathematics, p. 51-70. Vaasa: Faculty of education, Åbo Akademi University.

Lindenskov, L. & Weng P. (2013). *Matematikvanskeligheder – Tidlig intervention.* Dansk Psykologisk Forlag.

Lindenskov, L., Tonnesen, P. B., Weng, P., Østergaard, C. H. (in press). Theories to be combined and contrasted: Does the context make a difference? Early intervention programmes as case. In: K.Krainer & N. Vondrová (Eds.). *The proceedings CERME 9* – to be provided openly on the HAL archives website (https://hal.archives-ouvertes.fr/)

Mellin-Olsen, S. (1991). *Hvordan tenker lærere om matematikkundervisning?* [How do teachers think about mathematics teaching?] Landås: Bergen lærerhøgskole.

Mortimore, P., Evans, M. D., Laukkanen, R. & Valijarvi, J. (2004). *OECD-rapport om grundskolen i Danmark – 2004.* Uddannelsesstyrelsens temahæfte nr. 5. København: Undervisningsministriet. Kan hentes på: pub.uvm.dk/2004/oe½cd (september 2012)

Skovsmose, O. (2001). Landscapes of investigation. *Zentralblatt für Didaktik der Mathematik 33,* 1, 123-132

PISA 2009, Denmark:
http://uvm.dk/Uddannelser/Folkeskolen/Tosprogede/~/media/UVM/Filer/Udd/Folke/PDF11/111020_pisa_resultatrapport_2009.ashx

PISA 2012, Denmark:
http://www.uvm.dk/~/media/UVM/Filer/Udd/Folke/PDF13/Dec/131203%20PISA%20Rapport%20WEB.PDF

The Salamanca statement and framework for action on special needs education (1994):
http://www.unesco.org/education/pdf/SALAMA_E.PDF

Undervisningsministeriet (2013). Aftaleteksten.
http://www.uvm.dk/~/media/UVM/Filer/Udd/Folke/PDF13/130607%20Aftaleteksten.ashx

Undervisningsministeriet (2014). Fælles Mål Matematik.
http://www.emu.dk/omraade/gsk-l%C3%A6rer/ffm/matematik

Undervisningsministeriet (2009) Undervisningsvejledning for faget matematik.
http://www.uvm.dk/Service/Publikationer/Publikationer/Folkeskolen/2009/Faelles-Maal-2009-Matematik/Undervisningsvejledning-for-faget-matematik