What cognitive factors influence the development of mathematical difficulties?

Ulf Träff

Linköping University

Sweden

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Different terms or constructs related to low performance in mathematics?

Mathematical difficulties (general term)

Mathematical learning disability

Mathematical learning disorder (DSM V; Diagnostic and Statistical Manual of Mental Disorders, 2013)

Developmental Dyscalculia (acalculia)

Mathematical learning disability, disorder, and dyscalculia are used synonymous.

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Different terms or constructs related to low performance in mathematics?

Mathematical learning disability or disorder, and dyscalculia:

A neurodevelopmental disorder with a biological origin that affects cognitive functioning and mathematical ability/learning (DSM V).

It is a disorder within the individual.

Mathematical difficulties (general term) refers to poor mathematical performance that may be due to many different causes (inadequate education, social problems, poor language and reading skills).

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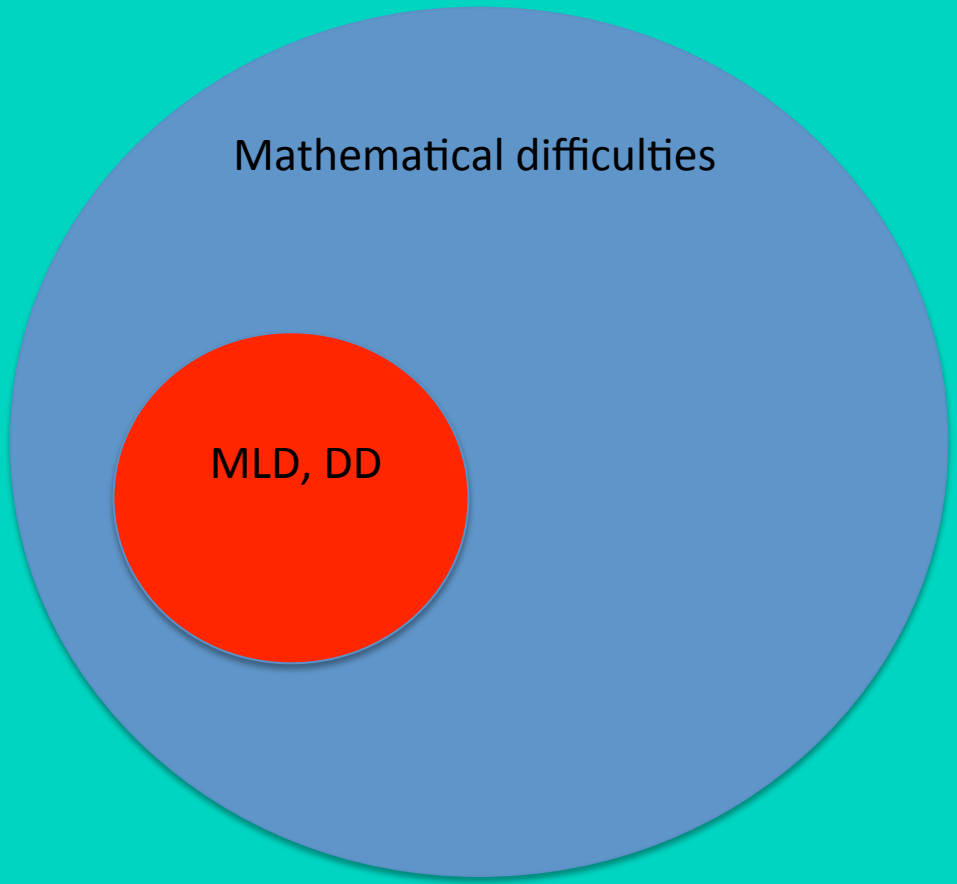
Berch & Mazzocco 2007, ,
Geary 2013; Price & Ansari 2013



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Definitions

ICD-10 (International Classification of Diseases-10)

Specific disorder of arithmetical skills
This disorder involves a specific impairment in arithmetical skills, which is not solely explicable on the basis of general **mental retardation or of grossly inadequate schooling.** The deficit concerns mastery of basic computational skills of addition, subtraction, multiplication, and division (rather than of the more abstract mathematical skills involved in algebra, trigonometry, geometry, or calculus).

Definitions DSM V (American Psychiatric Association. (2013). Diagnostic and statistical manual of mental disorders (5th ed.). Arlington, VA: American Psychiatric Publishing)

Diagnostic criteria

A. Difficulties learning and using academic skills, as indicated by the presence of at least one of the following **symptoms** that have **persisted for at least 6 months**, despite the **provision of interventions** that target those difficulties. **Persistent difficulties-Restricted progress in learning!**

5. Difficulties mastering number sense, number facts, or calculation (e.g., has poor understanding of numbers, their magnitude, and relationships; counts on fingers to add single-digit numbers instead of recalling the math fact as peers do; gest lost in the midst of arithmetic computation and may switch procedures).
6. Difficulties with mathematical reasoning (e.g., has severe difficulty applying mathematical concepts, facts, or procedures to solve quantitative problems).

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Definitions DSM V

Diagnostic criteria

B. **The affected academic skills are substantially and quantifiably below those expected for the individual's chronological age**, and cause significant interference with academic or occupational performance, or with activities of daily living, as confirmed by individually administered standardized achievement measures and comprehensive clinical assessment. For individuals age 17 years and older, a documented history of impairing learning difficulties may be substituted for the standardized assessment. **Well below average for age** (i.e., **at least 1.5 standard deviations (SD) below the population mean for age, below the 7th percentile)**

Definitions DSM V

Diagnostic criteria

D. The learning difficulties are not better accounted for by intellectual disabilities, uncorrected visual or auditory acuity, other mental or neurological disorders, psychosocial adversity, lack of proficiency in the language of academic instruction, or inadequate educational instruction.

It is a specific learning disorder that excludes all these explanations.

Cognitive factors that support children's learning of basic arithmetic.

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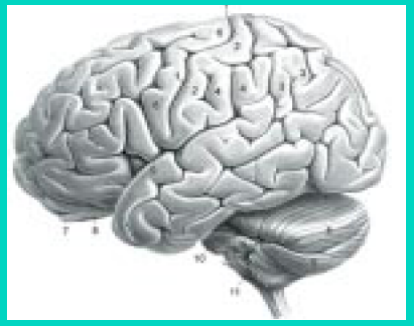
Educational instructions

EARLY MATHEMATICAL SKILLS
-**Arithmetic**
-Measurement
-Geometry

BEHAVIOUR

Specific number skills	General Cognitive skills
Place-value & base-10 system	Reading skills
Digit comparison	Problem solving skill
Arabic number code	Executive functions
Enumeration	Working memory
Counting (-words)	Long-term memory
Verbal number code	Language
Subitizing	Logic
Magnitude estimation	Spatial skills
Magnitude comparison	Processing speed
Core number system	Attention

COGNITION



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Domain specific number abilities

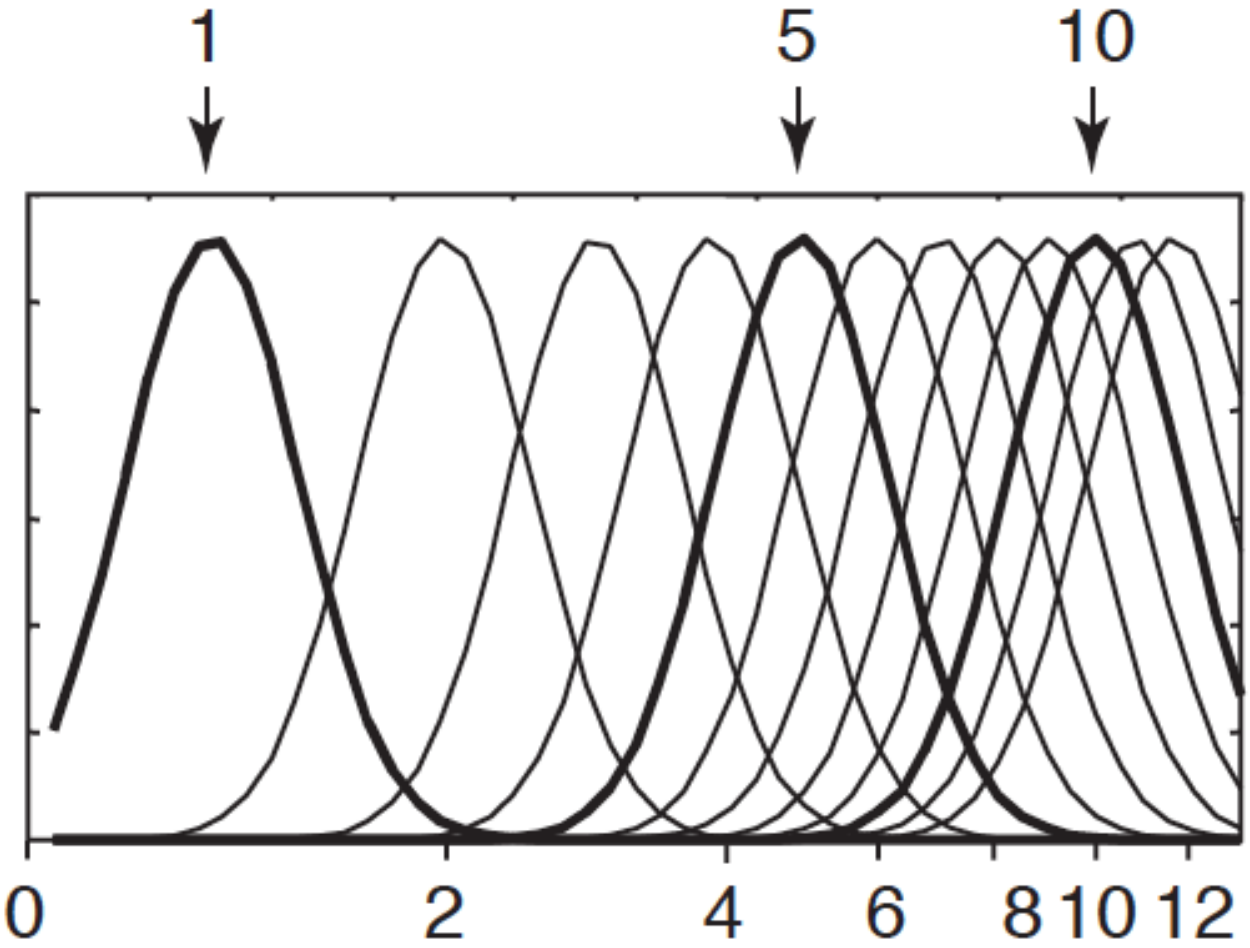
Humans are born with a core number representation system.

Two different accounts

The approximate number system (ANS), represents large, approximate numerosities on a spatially oriented mental number line (Dehaene et al., 1993; Dehaene, 1997; Feigenson, Dehaene & Spelke, 2004).

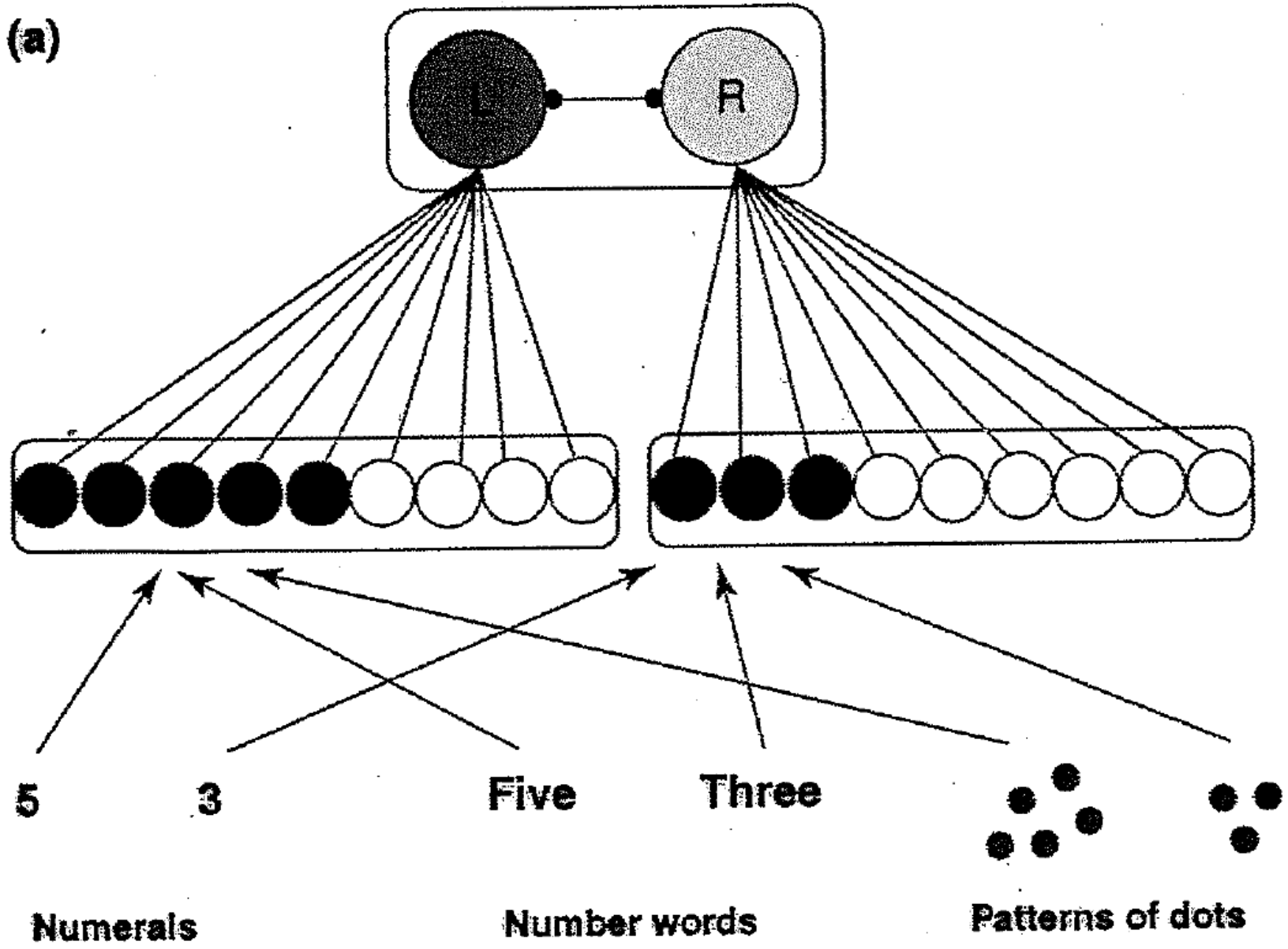
The ability to represent numerosities in a symbolic, exact fashion develops through mapping the number words and digits onto the ANS (De Smedt & Gilmore, 2011; Feigenson et al., 2004; Mundy & Gilmore, 2009; Piazza, 2010; Wilson & Dehaene, 2007).

Logarithmic model with fixed variability



Domain specific number abilities

According to the numerosity-coding theory are numerosity, and later on number words and digits, mapped onto an internal code that represents each numerosity as a set of discrete neurons or elements. (Butterworth, 2010; Zorzi et al., 2005).


(a)
L
R
5
3
Five
Three
Numerals
Number words
Patterns of dots

Domain specific number abilities

Both accounts state (Butterworth, 1999; Feigenson et al., 2004; Piazza, 2010) that:

-the core number system is critical for further development, as it enables fast-track learning (with normal environmental stimulation).

-it constitutes the foundation for the acquisition and development of the symbolic number system used for counting and arithmetic.

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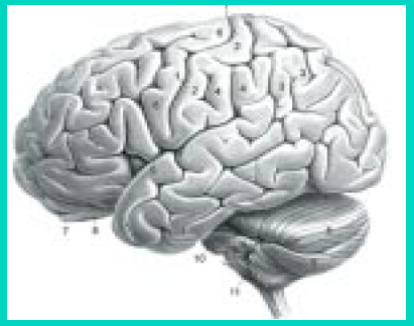

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Research show that both general cognitive abilities and number processing abilities underlying arithmetic skills in children (Fuchs et al., 2010a; 2010b; Geary, 2011; Martin Cirino, Sharp, & Barnes, 2014; Passolunghi, & Lanfranchi, 2012; Träff, 2013; Östergren & Träff, 2013).

What happens if one or more of the general and number processing functions do not function properly?

Mathematical difficulties
Or
Mathematical learning disability/disorder/dyscalculia?

Why is math so hard for some children and adults (Berch & Mazzocco, 2007)?

Different theoretical accounts of mathematical learning disorder/Developmental dyscalculia have been proposed.

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The defective ANS account

MLD/DD is due to a deficit in the innate approximate number system responsible for representing large and approximate numbers on a spatially oriented mental number line (Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010).

The precision and discriminability of the ANS in children with MD/DD is poor--poor acuity (sharpness)

Problems with non-symbolic and symbolic number processing.

The defective numerosity-coding account

This account states that MLD/DD in children is due to a deficit in recognizing, and representing exact numerosities (Butterworth,2010; Iuculano, Tang, Hall & Butterworth, 2008; Zorzi, Stoianov & Umilta, 2005).

An internal code that represents each numerosity as a set of discrete neurons or elements

The defective numerosity-coding account

This account states that MLD/DD in children is due to a deficit in the internal code that we used to recognize, and represent exact numerosities (Butterworth,2010; Iuculano, Tang, Hall & Butterworth, 2008; Zorzi, Stoianov & Umilta, 2005).

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The access deficit hypothesis

MLD/DD is not related to problems with processing of numerosities, but rather in accessing magnitude information from symbols (i.e., number words, digits) **(**Rousselle & Noël, 2007)**.**

A connection deficit between the symbolic number system and the innate core number magnitude system.

Problems with symbolic number processing, but not with non-symbolic number processing.

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Which number is the numerically larger one?

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The hypothesis of general cognitive deficits

Children with MLD have a deficit in their underlying cognitive system that prevents them from developing age-adequate skills in mathematics (Geary, 2004).

Problems with working memory, short-term memory, executive functions, processing speed, semantic long-term memory etc.

A number of studies show that children with **MATHEMATICAL DIFFICULTIES** have general cognitive deficits (e.g., Andersson & Lyxell, 2007; Bull, Johnston & Roy, 1999; D'Amico & Passolunghi, 2009; Passolunghi & Cornoldi, 2008; Raghubar et al., 2010; Swanson et al., 2004; Van der Sluis, de Jong & van der Leij, 2004).

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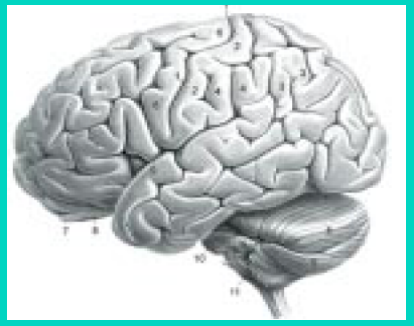

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Why is math so hard for some children and adults?

Specific number processing skills

-The defective ANS account

-The defective numerosity-coding account

-The access deficit hypothesis

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Mathematical learning disability/disorder

Dyscalculia (DSM V)

General Cognitive skills

The hypothesis of general cognitive deficits

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Mathematical difficulties

It is increasingly recognized that mathematical difficulties is a heterogeneous phenomenon, that might be caused by a number of underlying factors.

We have to make a distinction between different types of mathematical difficulties (Henik, Rubinsten & Ashkenazi, 2011; Kaufmann et al., 2013; Price & Ansari, 2013; Rubinsten & Henik, 2009).

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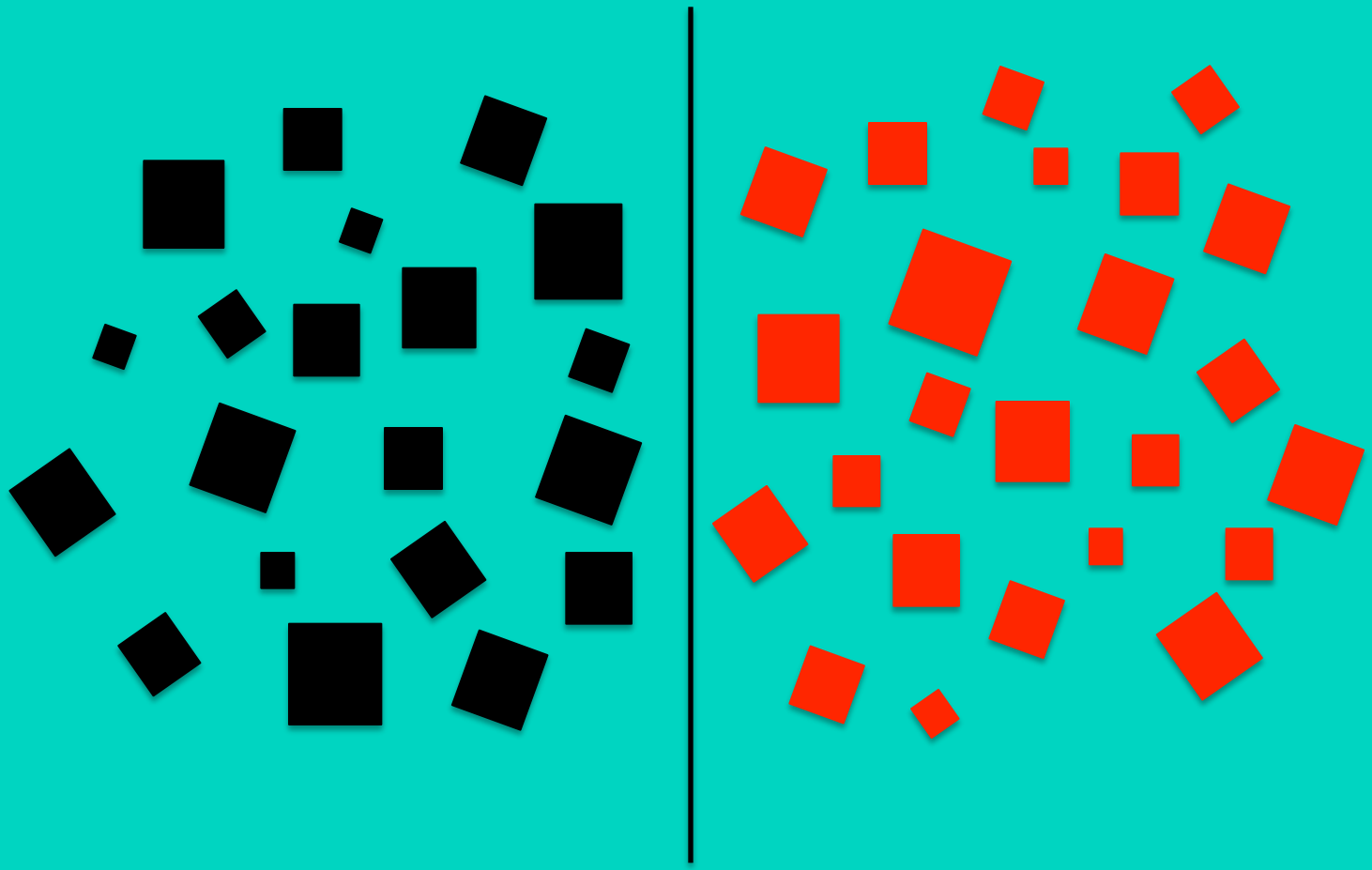
Comparison of the performance of DD/MLD and Control children on symbolic or non-symbolic number comparison.
A Core number deficit or an access deficit

References	Age (year)	Symbolic	Non-symbolic
De Smedt & Gilmore (2011)	6	DD<C	DD=C
Rousselle & Noël (2007)	7	DD<C	DD=C
Iuculano et al. (2008)	8–9	DD<C	DD=C
Landerl and Kölle (2009)	8–9–10	DD<C	DD=C
Landerl et al. (2009)	8–9–10	DD<C	DD<C
Piazza et al. (2010)	10	–	DD<C
Mussolin et al. (2010)	10–11	DD<C	DD<C
Price et al. (2007)	12	–	DD<C
Mazzocco et al. (2011)	14	–	DD<C
Skagerlund & Träff (2014)	10	DD=C	DD<C
Skagerlund & Träff (in press)	10–11–12	DD<C	DD<C
Olsson et al., (submitted)	9	DD<C	DD<C



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Which number is the numerically larger one?

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Why is math so hard for some children and adults?

Specific number processing skills

-The defective ANS account
-The defective numerosity-coding account
-The access deficit hypothesis

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Mathematical learning disability/disorder
Dyscalculia (DSM V)

General Cognitive skills

The hypothesis of general cognitive deficits

⬇

Mathematical difficulties

As all single core deficit accounts receive empirical support, a multiple deficits account has been proposed, stating that MLD/DD have multiple causes (Andersson & Östergren, 2012; Dowker, 2005; Jordan, Kaplan, & Hanich, 2002; Kosc, 1974; Mazzocco & Myers, 2003; Rubinsten & Henik, 2009; Temple, 1994; Wilson & Dehaene, 2007).

One child might suffer from a connection deficit whereas another might have a defective core number system, and third might suffer from an additive effect of these deficits.

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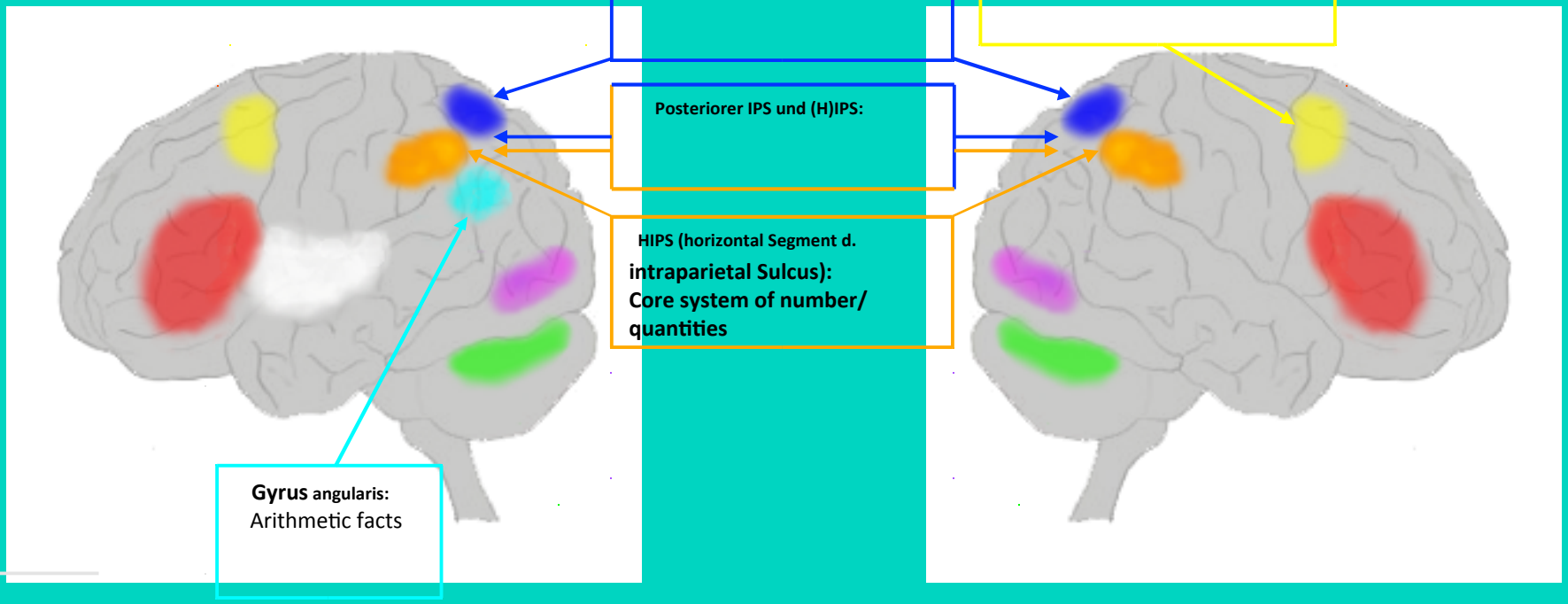
Brain areas that have been identified to be active during numerical and arithmetic processing

Parietal lobe



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Rykhlevskaia, E., Uddin, L.Q., Kondos, L., & Menon, V. (2009). Neuroanatomical correlates of developmental dyscalculia: combined evidence from morphometry and tractography. *Frontiers in human neuroscience, 3,* doi: 10.3389/neuro.09.051.2009.

Children and adults with dyscalculia exhibit both functional and structural deviations in the Intra-parietal sulcus compared to children and adults without dyscalculia

Functional deviation: Higher/lower brain activity (oxygen consumption) compared to individuals without dyscalculia

when performing number and arithmetic tasks.

Structural deviation: smaller volume of gray matter (brain tissue)

These deviations are connected to poorer performance.

Neuroimaging studies using diffusion tensor imaging (DTI) have explored white matter integrity in the brain in individuals with and without dyscalculia.

White matter development is an important aspect of brain maturation, reflecting connectivity between brain areas, and it is associated with learning (Ranpura et al., 2013).

Children with dyscalculia develop less amount of white matter in the frontal and parietal lobes.

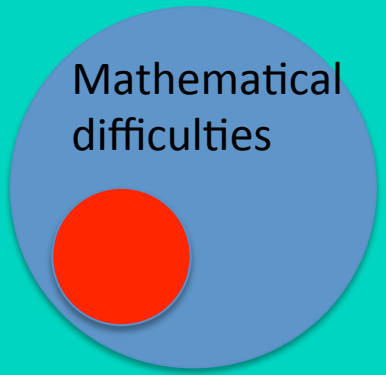
Dyscalculia may in part be due to faulty connections between the brain regions (frontal, parietal IPS) that are central to numerical and mathematical processing.

Summary and Conclusions

Mathematical difficulties is a heterogeneous phenomenon

A number of general cognitive and specific number processing factors influence the development of mathematical difficulties.

A distinction is made between mathematical difficulties and Mathematical learning disability, Mathematical learning Disorder, Dyscalculia



Different casual factors for mathematical difficulties and MLD/DD

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Summary and Conclusions

Both cognitive research and brain imaging research provide empirical support for a number processing deficit in MLD/DD

Brain imaging research support the assumption that MLD/DD have a biological origin.

Non of the number processing accounts obtain consistent and conclusive empirical support (i.e., a deficit in the core number system vs. the access deficit account).

Given the inconsistent findings and that MLD/DD is a heterogeneous condition (i.e., different profiles of mathematical deficits): a **multiple deficits account** is plausible.

Skagerlund and Träff (2015) found that children with general dyscalculia have impairment in the innate approximate number system, whereas children with arithmetic fact dyscalculia suffer from an access deficit.

When making decisions about interventions it is important to take into account profiles of number processing deficits and profiles of mathematical deficits.

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Thank you for your attention!

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